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ON THE CORRECTION OF EDDY FLUXES OF WATER VAPOUR AND TRACE GASES

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Abstract. The correction of eddy fluxes of water vapour and trace gases customarily called the Webb correction is presented and assessed. It is shown that the derivation of the conventional Webb correction equation is based on elements of a Boussinesq approximation. Such elements, however, should not be considered while any kind of flux correction equation is derived because flux correction equations that are, completely or partly, Boussinesq approximated violate conservation laws like the equation of continuity and the balance equations for water vapour and trace species postulated for turbulent systems. Furthermore, it is shown that density-weighted averaging procedures like those of Hesselberg (1926) and Swinbank (1951) that serve to completely satisfy all requirements associated with such conservation laws provide physically similar results. This is in contradiction to statements recently introduced into the literature. It is also shown that the correction effects based on such density-weighted averaging procedures are much smaller than those derived by the conventional Webb correction approach because they do not contain the effects of the eddy flux of sensible heat.

1. Introduction. Recently, Fuehrer and Friehe (2002) carefully analysed various approaches to the correction of eddy covariance terms, customarily called the Webb correction, that are published in the literature during the last 25 years (e.g., Webb and Pearman 1977, Bakan 1978, Smith and Jones 1979, Webb et al. 1980, Bernhardt and Piazena 1988, Kramm et al. 1995a, Kramm and Meixner 2000). This kind of flux correction plays an appreciable role in determining the exchange of trace gases like ozone, O_3, and carbon dioxide, CO_2, between the atmosphere and the vegetation-soil system because it can cause an enhancement of the vertical component of an eddy flux density (hereafter simply called an eddy flux) of matter only on the basis of so-called conservation laws like the equation of continuity and the local balance equations of water vapour and trace constituents derived for a turbulent system like the atmospheric surface layer (hereafter abbreviated by ASL). Recently, Massman and Lee (2002), Liebethal and Foken (2003), and Liu (2005) again discussed the significance of this kind of flux correction on the basis of their water vapour, H_2O, and CO_2 eddy flux measurements. Liebethal and Foken (2003) also analysed the method of Bernhardt and Piazena (1988), not considered in the review of Fuehrer and Friehe (2002). In contrast to that of Fuehrer and Friehe (2002), the contributions of Massman and Lee (2002), Liebethal and Foken (2003), and Liu (2005) are not self-consistent. The reasons for that can be summarised as follows: The conventional Webb correction was used to correct the water vapour flux. Furthermore, in their contribution, Massman and Lee (2002) and Liu (2005) considered the condition of an incompressible flow, $\nabla \cdot \mathbf{v} = 0$, in deriving flux correction equations, which has the following consequences: The instantaneous wind vector, $\mathbf{v}$, is spatially constant. Furthermore, expressing $\mathbf{v}$ by its mean value and the deviation from that, i.e., $\mathbf{v} = \overline{\mathbf{v}} + \mathbf{v}'$, and subsequent averaging of $\nabla \cdot \mathbf{v} = 0$ yields $\nabla \cdot \overline{\mathbf{v}} = 0$ because $\overline{\mathbf{v}'} = 0$. Here, ∇ is the nabla operator (also called the del operator), the overbar denotes the conventional Reynolds mean, and a prime characterises the departure from that. Because of $\nabla \cdot \mathbf{v} = 0$ and $\nabla \cdot \overline{\mathbf{v}} = 0$ it also follows that $\nabla \cdot \mathbf{v}' = 0$, i.e., not only the instantaneous wind vector is spatially constant, but also the mean wind vector and the fluctuating one. Moreover, if we assume horizontally homogeneous conditions as usually considered within the framework of the conventional Webb correction, i.e., $\partial u/\partial x = \partial v/\partial y = 0$, we will obtain

$$\frac{\partial w}{\partial z} = 0 \Rightarrow w = \text{const.} \tag{1.1}$$

Here, u, v, and w are the components of the wind vector in the horizontal (x and y) direction and in the vertical (z) direction, respectively. As at a rigid surface the vertical wind component vanishes, this constant must be equal to zero. The same is true in the cases of $\overline{w}$ and w' (e.g., Monin and Yaglom 1971). Since the total flux of a trace constituent with the partial density ρ_c is given by (e.g., Webb et al. 1980)

$$F_c = \overline{w\,\rho_c}, \tag{1.2}$$

one would obtain

$$F_c = \overline{w\,\rho_c} = 0, \tag{1.3}$$

i.e., over a rigid surface the total flux of a trace species would always be equal to zero. Consequently, the condition of an incompressible flow, $\nabla \cdot \mathbf{v} = 0$, is unsuitable and must not be used in manipulating local balance equations when flux correction equations are derived. In addition, Massman and Lee (2002) and Liu (2005) also used an equation of continuity for moist air that is not in agreement with the local formulation of the conservation of mass because it contains a source/sink term (the correct one is given by Eq. (3.1) of this paper). This is clearly a violation of one of the most important conservation laws. Also their quantity $\overline{\omega} = \overline{\rho_c}/\overline{\rho}$ is inaccurate because averaging $\omega = \rho_c/\rho$ yields $\overline{\omega} = (\overline{\rho_c} - \overline{\omega'\rho'})/\overline{\rho}$ (see, e.g., Eqs. (A3) and (A6) in Liu 2005). Here, ρ is the density of moist air. Ignoring the covariance term $\overline{\omega'\rho'}$ is only permitted within the framework of the so-called Boussinesq approximation. This, however, is not reasonable in deriving flux correction equations. Thus, the derivation of their flux correction equations is not self-consistent. Moreover, the contribution of Liebethal and Foken (2003) to the method of Bernhardt and Piazena (1988) is notably lacking because the governing conservation laws are inconsistently applied to evaluate this flux correction approach.

From a physical perspective, the conventional Webb correction also leads to results that seem to be doubtful. Based on their direct measurements of eddy fluxes of ozone and sensible heat performed over a desert area, Güsten et al. (1996), for instance, found that under very dry conditions (that lead to large Bowen ratios) the Webb correction would contribute to up to 40 per cent of the measured ozone fluxes. (These authors, for instance, were guided by a referee of their manuscript to correct their ozone eddy flux data according to Webb et al. 1980, H. Güsten 1996, personal communication.) Flux correction effects of up to 40 per cent are much larger than those estimated by Kramm et al. (1995a) and illustrated here in Figure 1. Since at the earth's surface, wind and density fluctuations vanish, such flux correction effects would imply an appreciable variation of the ozone eddy flux with height between the earth's surface and the measuring level owing to turbulence. Thus, as pointed out by Kramm and Meixner (2000), turbulence would act like an ozone net source.

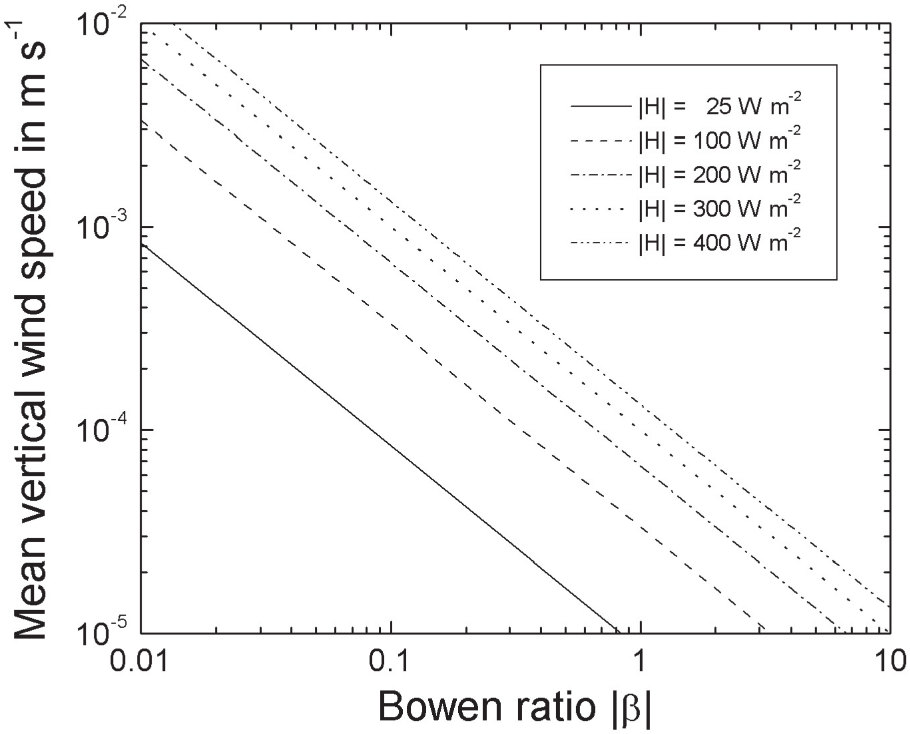

Fig. 1. The mean vertical wind component, $|\hat{w}|$, plotted against the Bowen ratio, $|\beta|$, for values of the eddy flux of sensible heat, $|H|$, (with reference to Kramm et al. 1995a). Note that this diagram comprises the data of Table 1 in Webb et al. (1980). For the purpose of comparison: A flow parallel to a slope of 0.057 degree (with respect to the horizontal direction) would generate a mean vertical wind component of 0.001 ms^{-1}, when a mean wind velocity of 1.00 ms^{-1} is adopted

Liebethal and Foken (2003), for instance, reported that during the EBEX-2000 experiment the Bowen ratio $\beta = H/E$ was extremely low (typical value during daytime: $\beta = 0.05$), even though the net radiation was very high (up to 700 W m^{-2}). Here, H is the eddy flux of sensible heat, and E is the eddy flux of latent heat. It seems that during this field campaign the dominant flux correction term was the water vapour flux rather than the sensible heat flux. If we assume that the maximum of the eddy fluxes of latent heat amounts to 500 W m^{-2}, as found by Liebethal and Foken (2003; see their Figure 1) a typical value during daytime of $\beta = 0.05$ would provide an eddy flux of sensible heat of 25 W m^{-2}. Liebethal and Foken (2003) also reported that the Webb correction of the CO_2 eddy flux amounts to 20 to 30 per cent. They also wrote: "Notice that the correction according to Kramm et al. (1995a) is much smaller and not dependent on the Bowen ratio. However, this method is based on a different averaging technique for turbulent parameters". Liu (2005) reported that the equations developed by him led to increased estimates of CO_2 uptake by the ecosystem, a black spruce forest in Interior Alaska, during the day of up to 20 per cent, and decreased estimates of CO_2 respiration by the ecosystem during the night (approximately 4 per cent). Thus, turbulence would also act like a CO_2 net source/sink because the CO_2 eddy flux would also appreciably vary with height. The attempt of Liebethal and Foken (2003) and Liu (2005) to correct the latent heat flux by correcting the H_2O eddy flux, however, seems to be awkward because such a flux correction is not in agreement with the conservation law for water vapour. Note that there is only a conservation law for water vapour, but not for latent heat. In other words: These authors corrected the H_2O eddy flux by the H_2O eddy flux, as already suggested by Webb et al. (1980) (see their Eq. (25)), Businger (1986) and many others. This is in complete contradiction to the results of Kramm et al. (1995a) as well as Kramm and Meixner (2000). Based on their exact derivations using Hesselberg's (1926) averaging procedure, these authors found that the H_2O eddy flux is invariant with height, i.e.,turbulence does not further affect it.

Nevertheless, huge flux correction effects as found by Güsten et al. (1996), Liebethal and Foken (2003), Liu (2005) and many other authors require a special attention because such flux correction estimates would appreciably affect the budgets of trace species like CO_2 (with exception of H_2O the most important "greenhouse" gas) and O_3 which might play a notable role in global climate change. Therefore, an overall assessment of this kind of flux correction seems to be indispensable.

In the following we will discuss the conventional Webb correction again, where especially the prerequisites and assumptions are pointed out on which this flux correction method is based. In Chapter 2, it is shown that the conventional Webb correction is based on elements of the Boussinesq approximation. As analysed in Chapter 3, such elements, however, must not be considered, when any kind of flux correction approach is derived because flux correction schemes which are, more or less, Boussinesq approximated violate the conservation laws mentioned above. Since Webb et al. (1980) also ignored the governing principles of conservation laws in deriving their flux correction formulae, it is shown in Chapter 4 that fulfilling the requirements associated with such conservation equations leads to different flux correction equations. This analysis is performed using the two different density-weighted averaging procedures, namely that of Hesselberg (1926) and that of Swinbank (1951), where the results are compared with each other. As argued by several authors (e.g., Hesselberg 1926, Ertel 1943, Montgomery 1948, 1954, van Mieghem 1949, 1951, 1973, Miller 1951, Swinbank 1951, Eliassen and Kleinschmidt 1957, Favre 1958, 1965, Bernhardt 1964, 1965, 1972, Herbert 1975, Libby and Williams 1980, Pichler 1984, Cox 1995, Dutton 1995, Kramm et al. 1995a, Thomson 1995, Kramm and Meixner 2000), such density-weighted averages are the common way to define averages in studies of highly compressible flows, probably the most natural way to define averages (for a historical review of density-weighted averaging procedures, see Lumley and Yaglom 2001). Density-weighted averages completely satisfy all requirements associated with conservation laws for turbulent systems. Hesselberg's (1926) averaging procedure was already used by Kramm et al. (1995a) as well as Kramm and Meixner (2000) to estimate flux correction effects. Swinbank's (1951) averaging calculus was strictly employed by Bernhardt (1964, 1965, 1972) as well as Businger and Deardorff (1968). Bernhardt and Piazena (1988) used it for estimating flux correction effects, too. It is shown that both density-weighted averaging procedures provide physically similar correction effects. Usually, they are much smaller than those provided by conventional Webb correction approaches. In Chapter 5, approximated equations for practical purposes are presented and discussed.

2. Assessing the conventional Webb correction. Webb and Pearman (1977), Bakan (1978), Smith and Jones (1979), and Webb et al. (1980), Leuning et al. (1982), Bernhardt and Piazena (1988), among others, suggested that in the case of trace constituents, such as CO_2, whose average concentrations are very large compared to the concentration fluctuations, the measurements of eddy fluxes in the ASL require corrections for air density fluctuations. Such density fluctuations may occur, for instance, due to water vapour fluctuations.

Following Webb et al. (1980) and similar treatments, the vertical flux component of a trace constituent is given by (see also Eq. (1.2))

$$F_c = \overline{w\,\rho_c} = \overline{w}\,\overline{\rho_c} + \overline{w'\rho_c'}. \tag{2.1}$$

Webb et al. (1980) argued that in the case when an eddy flux of sensible heat or water vapour exists, a non-zero mean vertical wind speed, $\overline{w}$, may occur owing to the correlated fluctuations of w and the partial density of dry air, ρ_a. Webb and Pearman (1977) as well as Webb et al. (1980) assumed that the mean vertical flux of the dry air constituent should be zero, i.e.,

$$\overline{w\rho_a} = 0. \tag{2.2}$$

Since Bakan (1978) argued that $\overline{w\rho} = 0$, Leuning et al. (1982) stated that the premise (2.2) is *undoubtedly* correct because Bakan's assumption would lead to the contradiction $\overline{w\rho} = \overline{w\rho_a} + \overline{w\rho_v} = \overline{w\rho_v} \neq 0$ when there is evaporation. Here, ρ_v is the partial density of water vapour. Obviously, the argument of Leuning et al. (1982) already implies that the premise (2.2) is correct, i.e., this premise is justified in the sense of a vicious circle. Since, however, many gaseous trace species like CO_2 and O_3, contributing to the composition of dry air, are emitted and/or absorbed by the ground, the premise (2.2) cannot be entirely correct. Within the framework of the Webb correction, however, it appears to be plausible because Webb et al. (1980) assumed that $\rho_c \ll \rho_a + \rho_v$, and hence, $\rho = \rho_a + \rho_v + \rho_c \cong \rho_a + \rho_v$, even though ρ_c is a portion of ρ_a. Nevertheless, we have to recognise this premise; otherwise any further discussion of the Webb correction becomes fruitless because Eq. (2.2) is the most prominent prerequisite on which this flux correction approach is based. Consequently, trace species must be treated separately. This means that dry air (i.e., without trace species) is assumed as chemically inert, and its total flux may be considered as equal to zero. In these premises, Eq. (2.2) results in

$$\overline{w} = -\frac{\overline{w'\rho_a'}}{\overline{\rho_a}}. \tag{2.3}$$

Assuming that the partial density of trace species can be ignored, i.e., $\rho_c \ll \rho_a + \rho_v$, Webb et al. (1980) wrote (see their Eq. (4))

$$\frac{\rho_a}{m_a} + \frac{\rho_v}{m_v} = \frac{p}{RT}. \tag{2.4}$$

Here, m_a and m_v are the molecular weights of dry air (i.e., a computed value for the gas mixture dry air) and water vapour, respectively. The quantity $R \cong 8.314$ J K^{-1} mol^{-1} is the universal gas constant, and T is the absolute temperature. Webb et al. (1980) expanded $T^{-1} = (\overline{T} + T')^{-1}$ in powers of $T'/\overline{T}$ obtaining (see their Eq. (8a))

$$\frac{\rho_a}{m_a} + \frac{\rho_v}{m_v} = \frac{p}{R\overline{T}} \left\{ 1 - \frac{T'}{\overline{T}} + \left(\frac{T'}{\overline{T}}\right)^2 - \left(\frac{T'}{\overline{T}}\right)^3 + \cdots \right\}. \tag{2.5}$$

Averaging this Taylor series in the sense of Reynolds yields then (see the Eq. (8b) of Webb et al. 1980)

$$\frac{\overline{\rho_a}}{m_a} + \frac{\overline{\rho_v}}{m_v} = \frac{p}{R\overline{T}} \left\{ 1 + \frac{\overline{T'^2}}{\overline{T}^2} - \frac{\overline{T'^3}}{\overline{T}^3} + \cdots \right\}. \tag{2.6}$$

Since $\rho'_k = \rho_k - \overline{\rho_k}$, where k stands for a and v, one obtains by subtracting Eq. (2.6) from Eq. (2.5) (see Eq. (8c) of Webb et al. 1980)

$$\frac{\rho'_a}{m_a} + \frac{\rho'_v}{m_v} = \frac{p}{R\overline{T}} \left\{ -\frac{T'}{\overline{T}} + \frac{T'^2 - \overline{T'^2}}{\overline{T}^2} - \frac{T'^3 - \overline{T'^3}}{\overline{T}^3} + \cdots \right\}. \tag{2.7}$$

If the term $p/(R\overline{T})$ is eliminated by use of Eq. (2.6), we will obtain (see Eq. (9a) of Webb et al. 1980)

$$\rho'_a = -\mu\rho'_v - \overline{\rho_a}(1 + \mu r_v) \frac{\frac{T'}{\overline{T}} - \frac{T'^2 - \overline{T'^2}}{\overline{T}^2} + \frac{T'^3 - \overline{T'^3}}{\overline{T}^3} - \cdots}{1 + \frac{\overline{T'^2}}{\overline{T}^2} - \frac{\overline{T'^3}}{\overline{T}^3} + \cdots} \tag{2.8}$$

where $\mu = m_a/m_v$, and $r_v = \overline{\rho_v}/\overline{\rho_a}$. By ignoring the terms $T'^2/\overline{T}^2$ and $\overline{T'^2}/\overline{T}^2$ and corresponding higher-order terms, Webb et al. (1980) obtained (see their Eq. (9b))

$$\rho'_a \cong -\mu\rho'_v - \overline{\rho_a}(1 + \mu r_v)\frac{T'}{\overline{T}}. \tag{2.9}$$

Introducing Eq. (2.9) into Eq. (2.3) yields then

$$\overline{w} = \mu \frac{\overline{w'\rho'_v}}{\overline{\rho_a}} + (1 + \mu r_v)\frac{\overline{w'T'}}{\overline{T}}. \tag{2.10}$$

The covariance terms $\overline{w'\rho'_v}$ and $\overline{w'T'}$ may be expressed in terms of the sensible heat flux, $H \cong c_p\overline{\rho}\,\overline{w'T'}$, and the Bowen ratio $\beta = H/E$, where c_p is the specific heat at constant pressure, $E = L_v\,\overline{w'\rho'_v}$ is the latent heat flux, and L_v is the specific heat of vaporization, to discuss the relative importance of both flux correction terms, i.e.,

$$\overline{w} \cong \frac{H}{c_p\overline{\rho}} \left(\mu \frac{\overline{\rho}}{\overline{\rho_a}} \frac{c_p}{L_v\beta} + \frac{1}{\overline{T}}(1 + \mu r_v) \right). \tag{2.11}$$

Equation (2.11) shows that the flux correction estimate mainly depends of the sensible heat flux; whereas the effects owing to the density fluctuations of water vapour are negligible, except when $|\beta|$ is small (Kramm et al. 1995a).

It is well known that the Bowen ratio can be positive or negative (in the case of $\beta = 0$ we have to consider Eq. (2.10) because Eq. (2.11) becomes unpredictable). Thus, in the case of a negative Bowen ratio there is the possibility that the mean vertical wind speed established by Eq. (2.11) vanishes. If we adopt, for instance, $\overline{T} = 300$ K and $L_v = 2.5 \cdot 10^6$ J kg^{-1}, we will obtain: $\beta = -0.12$.

As argued by Webb et al. (1980) and many other authors, the mean vertical wind speed is only related to covariance terms $\overline{w'\rho_v'}$ and $\overline{w'T'}$. Equation (2.10) serves to replace $\overline{w}$ in Eq. (2.1). One obtains (see Eq. (24) of Webb et al. 1980)

$$F_c = \overline{w'\rho_c'} + \mu \frac{\overline{\rho_c}}{\overline{\rho_a}} \overline{w'\rho_v'} + (1 + \mu r_v) \frac{\overline{\rho_c}}{\overline{T}} \overline{w'T'}. \quad (2.12)$$

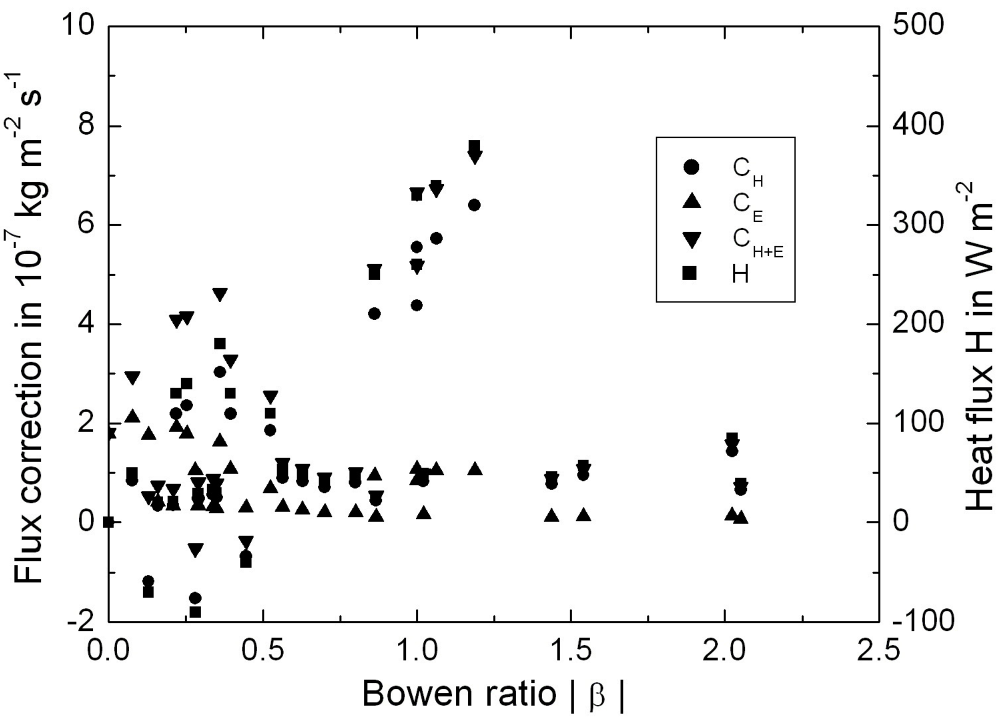

Fig. 2. Flux correction effects inferred from the vertical eddy fluxes of sensible, C_H, and latent heat, C_E, plotted against the Bowen ratio, $|\beta|$ (data taken from Webb et al. 1980, Table 1). The "total" flux correction, C_{H+E}, and the eddy flux of sensible heat, H, are also shown. Note that C_E should only be considered as the total flux correction.

As illustrated in Figure 2, in most cases the heat flux term dominates the flux correction. For the water vapour flux, Q, Webb et al. (1980) suggested (see their Eq. (25))

$$Q = (1 + \mu r_v) \left\{ \overline{w' \rho_v'} + \frac{\overline{\rho_v}}{\overline{T}} \overline{w' T'} \right\} . \tag{2.13}$$

As over a desert area the water vapour flux is negligible, it is not surprising that Güsten et al. (1996) obtained a large percentage of the flux correction. Considering for a moment Eq. (A10) of this paper and introducing it into Eq. (2.3) would directly provide $\overline{w} = \overline{w'T'}/\overline{T}$ because all water vapour effects are negligible under such very dry conditions. Thus, as stated before, Eq. (2.2) is the most prominent prerequisite on which this kind of flux correction is based. However, no conservation law demands that Eq. (2.2) hold true, as already stated by Venkatram (1993).

The derivation of the flux correction equations (2.12) and (2.13), however, is not self-consistent. Expressing, for instance, the instantaneous pressure of air in Eq. (2.5) by $p = \overline{p} + p'$ yields

$$\frac{\rho_a}{m_a} + \frac{\rho_v}{m_v} = \frac{\overline{p}}{R\overline{T}} \left\{ 1 - \frac{T'}{\overline{T}} + \left(\frac{T'}{\overline{T}} \right)^2 - \left(\frac{T'}{\overline{T}} \right)^3 + \cdots \right\} + \frac{p'}{R\overline{T}} \left\{ 1 - \frac{T'}{\overline{T}} + \left(\frac{T'}{\overline{T}} \right)^2 - \left(\frac{T'}{\overline{T}} \right)^3 + \cdots \right\} . \tag{2.14}$$

Averaging this equation in the sense of Reynolds provides then

$$\frac{\overline{\rho_a}}{m_a} + \frac{\overline{\rho_v}}{m_v} = \frac{\overline{p}}{R\overline{T}} \left\{ 1 + \frac{\overline{T'^2}}{\overline{T}^2} - \frac{\overline{T'^3}}{\overline{T}^3} + \cdots \right\} + \frac{1}{R\overline{T}} \left\{ - \frac{\overline{p'T'}}{\overline{T}} + \frac{\overline{p'T'^2}}{\overline{T}^2} - \frac{\overline{p'T'^3}}{\overline{T}^3} + \cdots \right\} . \tag{2.15}$$

Consequently, the term $p/(R\overline{T})$ that occurs in Eq. (2.7) cannot be eliminated using Eq. (2.5) because $p/(R\overline{T}) \neq \overline{p}/(R\overline{T})$. This result is, clearly, in contradiction to Webb et al. (1980).

Subtracting Eq. (2.15) from Eq. (2.14) yields then

$$\left. \begin{aligned} \frac{\rho_a'}{m_a} + \frac{\rho_v'}{m_v} = {} & \frac{\overline{p}}{R\overline{T}} \left\{ - \frac{T'}{\overline{T}} + \frac{T'^2 - \overline{T'^2}}{\overline{T}^2} - \frac{T'^3 - \overline{T'^3}}{\overline{T}^3} + \cdots \right\} \\ & + \frac{p'}{R\overline{T}} \left\{ 1 - \frac{T'}{\overline{T}} + \left(\frac{T'}{\overline{T}} \right)^2 - \left(\frac{T'}{\overline{T}} \right)^3 + \cdots \right\} \\ & - \frac{1}{R\overline{T}} \left\{ - \frac{\overline{p'T'}}{\overline{T}} + \frac{\overline{p'T'^2}}{\overline{T}^2} - \frac{\overline{p'T'^3}}{\overline{T}^3} + \cdots \right\} \end{aligned} \right\} \tag{2.16}$$

or

$$\left. \begin{aligned} \rho_a' = -\mu \rho_v' \; & + \frac{\overline{p}}{R_a\overline{T}} \left\{ \frac{p'}{\overline{p}} - \frac{T'}{\overline{T}} + \frac{T'^2 - \overline{T'^2}}{\overline{T}^2} - \frac{T'^3 - \overline{T'^3}}{\overline{T}^3} + \cdots \right\} \\ & + \frac{1}{R_a\overline{T}} \left\{ - \frac{p'T' - \overline{p'T'}}{\overline{T}} + \frac{p'T'^2 - \overline{p'T'^2}}{\overline{T}^2} - \frac{p'T'^3 - \overline{p'T'^3}}{\overline{T}^3} + \cdots \right\} \end{aligned} \right\} \tag{2.17}$$

where $R_a = R/m_a$ is the computed "gas constant" of dry air. If we introduce Eq. (2.17) into Eq. (2.3), we can finally derive

$$\left.\begin{aligned}\overline{w} = \frac{\mu}{\overline{\rho_a}}\overline{w'\rho_v'} &- \frac{\overline{p}}{\overline{\rho_a}R_a\overline{T}}\left\{\frac{\overline{w'p'}}{\overline{p}} - \frac{\overline{w'T'}}{\overline{T}} + \frac{\overline{w'T'^2}}{\overline{T}^2} - \frac{\overline{w'T'^3}}{\overline{T}^3} + \cdots\right\} \\ &- \frac{1}{\overline{\rho_a}R_a\overline{T}}\left\{-\frac{\overline{w'p'T'}}{\overline{T}} + \frac{\overline{w'p'T'^2}}{\overline{T}^2} - \frac{\overline{w'p'T'^3}}{\overline{T}^3} + \cdots\right\}\end{aligned}\right\}. \tag{2.18}$$

Like in the Eqs. (76) to (79) of Fuehrer and Friehe (2002), pressure fluctuation terms occur in Eq. (2.18). Based on the argument that T'^2 is less well correlated with w' than is T', Webb et al. (1980) ignored, for instance, the term $\overline{w'T'^2}/\overline{T}^2$ and corresponding higher-order terms relative to the term $\overline{w'T'}/\overline{T}$. However, they also mentioned that this becomes less true for skewed T' distributions occurring when convective conditions prevail. Nevertheless, if we accept this argument and, in addition, ignore the third term of the right-hand side of Eq. (2.18), we will obtain

$$\overline{w} = \frac{\mu}{\overline{\rho_a}}\overline{w'\rho_v'} - \frac{\overline{p}}{\overline{\rho_a}R_a\overline{T}}\left(\frac{\overline{w'p'}}{\overline{p}} - \frac{\overline{w'T'}}{\overline{T}}\right). \tag{2.19}$$

Introducing this equation into Eq. (2.1) yields the following approximation.

$$F_c = \overline{w'\rho_c'} + \mu\frac{\overline{\rho_c}}{\overline{\rho_a}}\overline{w'\rho_v'} - \frac{\overline{\rho_c}}{\overline{\rho_a}}\frac{\overline{p}}{R_a\overline{T}}\left(\frac{\overline{w'p'}}{\overline{p}} - \frac{\overline{w'T'}}{\overline{T}}\right). \tag{2.20}$$

In comparison with Eqs. (2.10) to (2.13), we may conclude that the term $\overline{w'p'}/\overline{p}$ was ignored by Webb et al. (1980), as already stated by Kramm et al. (1995a) and Fuehrer and Friehe (2002). As demonstrated in Appendix A, ignoring pressure fluctuations is one of the elements of the Boussinesq approximation. Note that including the pressure transport term $\overline{w'p'}$ for correcting eddy covariance terms requires measurements of pressure fluctuations. Following, for instance, Wilczak et al. (1995), such measurements can be performed with a sufficient degree of accuracy.

The pressure transport term $\overline{w'p'}$ also plays a notable role in the local balance equation of turbulent kinetic energy (TKE). Assuming horizontally homogeneous conditions, as considered within the framework of the conventional Webb correction, the Boussinesq approximated form of this balance equation reads (e.g., Garratt 1994)

$$\frac{\partial\overline{e}}{\partial t} = -\overline{u'w'}\frac{\partial\overline{u}}{\partial z} - \overline{v'w'}\frac{\partial\overline{v}}{\partial z} + \frac{g}{\overline{\Theta}_v}\overline{w'\Theta_v'} - \frac{\partial}{\partial z}\left(\overline{w'e} + \frac{\overline{w'p'}}{\overline{\rho}}\right) - \overline{\varepsilon}. \tag{2.21}$$

Here, t is time, $e = (u'^2 + v'^2 + w'^2)/2$ is the TKE per unit mass, g is the acceleration of gravity, Θ_v is the virtual potential temperature, and $\overline{\varepsilon}$ is the mean dissipation of TKE. As pointed out by Garratt (1994), “in the strongly unstable surface layer, according to the observations of Högström (1990), the pressure transport term $\partial(\overline{w'p'})/\partial z$ is a significant source of TKE with $\overline{w'p'}$ being zero at a rigid surface and becoming increasingly negative with increasing height”. Högström's (1990) findings substantially agree with those of Wyngaard and Coté (1971), McBean and Elliot (1975), and Wilczak et al. (1995). Therefore, we must not expect that the pressure transport term is negligible under strongly unstable conditions.

3. The equation of continuity and the Boussinesq approximation. To assess flux correction procedures like the conventional Webb correction method from a physical perspective, it is indispensable to consider the governing conservation equations. If we employ the conventional Reynolds averaging procedure, the equation of continuity for a turbulent system will read

$$\frac{\partial \overline{\rho}}{\partial t} + \nabla \cdot (\overline{\rho \mathbf{v}}) = \frac{\partial \overline{\rho}}{\partial t} + \nabla \cdot (\overline{\rho}\,\overline{\mathbf{v}} + \overline{\rho' \mathbf{v}'}) = 0. \tag{3.1}$$

Here, the term $\overline{\rho}\,\overline{\mathbf{v}}$ is called the convective transport, and $\overline{\rho' \mathbf{v}'}$ is denoted as a non-convective transport. Equation (3.1) demonstrates that there is no production or destruction of the mean total mass within any given fixed volume (i.e., conservation of total mass on a local scale) with respect to a turbulent system. This means that a source/sink term, as introduced by Massman and Lee (2002) and Liu (2005) for the gas mixture moist air, is not in agreement with this governing equation. Assuming horizontal homogeneity and stationary conditions (also assumed by Webb et al. 1980, and many other authors) lead then to

$$\frac{\partial}{\partial z}(\overline{\rho w}) = \frac{\partial}{\partial z}(\overline{\rho}\,\overline{w} + \overline{\rho' w'}) = 0 \Rightarrow \overline{\rho w} = \overline{\rho}\,\overline{w} + \overline{\rho' w'} = \text{const.} \tag{3.2}$$

This constant has to be determined. If we express Eq. (2.2) by

$$\overline{w \rho_a} = \overline{w \rho} - \overline{w \rho_v} = 0, \tag{3.3}$$

we will obtain for this constant

$$\overline{w}\overline{\rho} + \overline{w' \rho'} = \overline{w}\overline{\rho_v} + \overline{w' \rho_v'} = \text{const.} \tag{3.4}$$

or

$$\overline{w} = \frac{1}{\overline{\rho_a}}(\overline{w' \rho_v'} - \overline{w' \rho'}) \tag{3.5}$$

Pressure fluctuations can be related to the mean pressure by (see Eq. (A7) of Appendix A),

$$\frac{p'}{\overline{p}} = \frac{\rho'}{\overline{\rho}} \frac{1}{1 + \frac{\overline{\rho' T_v'}}{\overline{T_v}}} + \frac{T_v'}{\overline{T_v}} \frac{1}{1 + \frac{\overline{\rho' T_v'}}{\overline{\rho}}} + \frac{\rho' T_v' - \overline{\rho' T_v'}}{\overline{\rho}\,\overline{T_v} \left(1 + \frac{\overline{\rho' T_v'}}{\overline{\rho}\,\overline{T_v}}\right)} \tag{3.6}$$

where T_v is the virtual temperature. If this equation is approximated in the sense of Boussinesq, i.e., all second moments containing air density fluctuations are ignored except in those terms expressing the effect of the gravity field $-\nabla\phi$ on the density fluctuations ρ' occurring in the mass field ρ (e.g., van Mieghem 1973, Holton 1979), and $\rho' T' \cong \overline{\rho' T'}$ is assumed (usually not true), we will obtain the classical simplification (e.g., Holton 1979, Businger 1982, Stull 1988)

$$\rho' = \overline{\rho} \left(\frac{p'}{\overline{p}} - \frac{T_v'}{\overline{T_v}} \right). \tag{3.7}$$

Introducing this equation into Eq. (3.5) yields

$$\overline{w} = \frac{1}{\overline{\rho_a}} \left\{ \overline{w' \rho_v'} - \overline{\rho} \left(\frac{\overline{w' p'}}{\overline{p}} - \frac{\overline{w' T_v'}}{\overline{T_v}} \right) \right\} = \frac{\overline{w' \rho_v'}}{\overline{\rho_a}} - (1 + r_v) \left(\frac{\overline{w' p'}}{\overline{p}} - \frac{\overline{w' T_v'}}{\overline{T_v}} \right). \tag{3.8}$$

Obviously, this equation has the same structure than Eq. (2.19). Both equations indicate that ignoring the pressure transport term, as done by Webb et al. (1980), is the most important assumption in deriving Eq. (2.10). In contrast to Eq. (14) of Webb et al. (1980), the expression (3.8) can be derived using the approximation (3.7). Consequently, it is not necessary to consider Taylor series and to neglect $T'/\overline{T}$-terms of the second order and higher orders.

Ignoring the pressure fluctuations in Eq. (3.7) that results in

$$\frac{\rho'}{\overline{\rho}} \cong -\frac{T_v'}{\overline{T_v}} \tag{3.9}$$

yields (see also Kramm et al. 1995a)

$$\overline{w} \cong \frac{1}{\overline{\rho_a}}\left(\overline{\rho_v' w'} + \overline{\rho}\frac{\overline{w'T_v'}}{\overline{T_v}}\right) = \frac{\overline{\rho_v' w'}}{\overline{\rho_a}} + (1 + r_v)\frac{\overline{w'T_v'}}{\overline{T_v}}, \tag{3.10}$$

i.e., also Eqs. (2.10) and (3.10) have the same structure. Again, we found that Webb et al. (1980) ignored the pressure transport term. The conventional Webb correction not only implies that several second moments containing air density fluctuations are ignored (leading to Eq. (3.7)), but also neglects the pressure transport term. A fluid that mainly fulfils these conditions is customarily called a Boussinesq fluid. Apparently, the conventional Webb correction is based on an inconsistent utilisation of the Boussinesq approximation leading to a notable violation of the conservation laws, as already demonstrated by Kramm et al. (1995a) and Kramm and Meixner (2000).

Another aspect of the conventional Webb correction has also been elucidated. Since the constant in Eq. (3.2) must also match the condition at $z = 0$, one may conclude, in agreement with Bakan (1978), that this constant is equal to zero because, as mentioned before, at a rigid surface the vertical wind component must vanish. Consequently, the statement of Leuning et al. (1982) that the assumption (2.2) is undoubtedly correct cannot be justified on the basis of the governing principles of the equation of continuity, as expressed by Eq. (3.2) for horizontally homogeneous conditions (another major prerequisite of Webb et al. 1980). Venkatram's (1993) statement mentioned before that no conservation law demands that Eq. (2.2) hold true is, therefore, entirely correct.

4. The use of density-weighted averaging procedures to correct trace gas fluxes. To guarantee that the equation of continuity and the balance equations for water vapour and trace species are serving as true conservation laws, we must not ignore terms that contain density and pressure fluctuations, as usually done in the case of a Boussinesq fluid. Alternatively, we may consider density-weighted averages, as suggested in the matter of flux correction by Kramm et al. (1995a) and Kramm and Meixner (2000). These authors used Hesselberg's (1926) density-weighted averaging calculus to completely satisfy all requirements associated with conservation laws. Since Webb et al. (1980) (see their Eq. (16)), Bernhardt and Piazena (1988) as well as Liebethal and Foken (2003) mentioned Swinbank's (1951) density-weighted averaging calculus, where Liebethal and Foken (2003) also tried to infer the contradictory results of their Webb correction attempt and those of Kramm et al. (1995a) to the use of different averaging procedures (see citation in Chapter 1), it is shown that in both cases of density-weighted averaging the flux correction estimates reflect the same physical behaviour. Note that the use of density-weighted averages in practise also requires measurements of pressure fluctuations.

Swinbank's (1951) density-weighted averaging calculus, for instance, yields

$$\overline{\rho\mathbf{v}\varphi} = \overline{\rho\mathbf{v}}\,\overline{\varphi} + \overline{(\rho\mathbf{v})'\varphi'}, \tag{4.1}$$

where φ stands for the wind vector, $\mathbf{v}$, the temperature, T, and the pressure p as well as the mass fractions $\varphi_k = \rho_k/\rho$ of dry air ($k = a$), water vapour ($k = v$), and trace species ($k = c$). Obviously, this averaging calculus is dealing with the fluctuations of the vector of momentum, rather than with the fluctuations of the wind vector (for more details, see Kramm and Meixner 2000). By considering this averaging calculus, the equation of continuity for a turbulent system reads

$$\frac{\partial\overline{\rho}}{\partial t} + \nabla\cdot(\overline{\rho\mathbf{v}}) = 0 \tag{4.2}$$

because, according to Eq. (4.1), $\overline{(\rho\mathbf{v})'}$ is equal to zero when $\varphi = 1$ is considered. Obviously, the equation of continuity holds its form, in complete contrast to Eq. (3.1). For the purpose of comparison: Hesselberg's (1926) averaging calculus provides (e.g., van Mieghem 1949, 1973, Herbert 1975, Picher 1984, Dutton 1995, Cox 1995, Kramm et al. 1995a, Kramm and Meixner 2000)

$$\overline{\rho\mathbf{v}\varphi} = \overline{\rho}\hat{\mathbf{v}}\hat{\varphi} + \overline{\rho\mathbf{v}''\varphi''} \tag{4.3}$$

so that the difference between the averaging procedure of Swinbank and Hesselberg can be expressed by (Montgomery 1954, Kramm and Meixner 2000)

$$\overline{(\rho\mathbf{v})'\varphi'} = \overline{\rho\mathbf{v}''\varphi''} + \hat{\mathbf{v}}\overline{\rho'\varphi'}. \tag{4.4}$$

Here, the roof denotes Hesselberg's (1926) density-weighted average, $\hat{\varphi} = \overline{\rho\varphi}/\overline{\rho}$, and a double prime the departure from that, where $\widehat{\varphi''} = 0$. By choosing $\varphi = 1$ in Eq. (4.3), we obtain for the equation of continuity for a turbulent system

$$\frac{\partial\overline{\rho}}{\partial t} + \nabla\cdot(\overline{\rho\mathbf{v}}) = \frac{\partial\overline{\rho}}{\partial t} + \nabla\cdot(\overline{\rho}\hat{\mathbf{v}}) = 0. \tag{4.5}$$

In complete contrast to Eq. (3.1), this equation of continuity holds its form, too. Furthermore, in both cases of density-weighted averages, the mean value of the kinetic energy can exactly be split into the kinetic energy of the mean motion and the mean value of the eddying motion, i.e.,

$$\frac{1}{2}\overline{\rho\mathbf{v}^2} = \frac{1}{2}\overline{\rho\mathbf{v}}\cdot\overline{\mathbf{v}} + \frac{1}{2}\overline{(\rho\mathbf{v})'\cdot\mathbf{v}'} \tag{4.6}$$

and (van Mieghem 1949, 1973, Pichler 1984, Kramm et al. 1995a)

$$\frac{1}{2}\overline{\rho\mathbf{v}^2} = \frac{1}{2}\overline{\rho}\hat{\mathbf{v}}^2 + \frac{1}{2}\overline{\rho\mathbf{v}''^2}. \tag{4.7}$$

This is impossible in the case of conventional Reynolds averaging because terms of the form $\overline{\rho'\mathbf{v}'}\cdot\overline{\mathbf{v}}$ and $\overline{\rho'\mathbf{v}'^2}$ occur that also contain density fluctuations. These terms have to be ignored, as customarily done in the case of a Boussinesq fluid. Another prominent advantage of the Hesselberg's (1926) averaging procedure is that the substantial derivative with respect to time of any property,

d/dt, can exactly be expressed by Euler's operator for the averaged turbulent flow of a compressible atmosphere called a Hesselberg fluid (Kramm and Meixner 2000),

$$\frac{d}{dt} = \frac{\partial}{\partial t} + \hat{\mathbf{v}} \cdot \nabla \tag{4.8}$$

Such a derivation is impossible in the case of conventional Reynolds averaging because an expression for Euler's operator similar to Eq. (4.8) can only be deduced when the density fluctuation terms $\partial(\overline{\rho' \varphi_k'})/\partial t$ and $\nabla \cdot (\overline{\mathbf{v}}\,\overline{\rho' \varphi_k'} + \overline{\rho' \mathbf{v}' \varphi_k'})$ are ignored as customarily done within the framework of the Boussinesq approximation. Also Swinbank's averaging procedure does not allow forming an exact Euler operator.

Let us consider the balance equations for dry air, water vapour, and trace constituents derived for a turbulent system on the basis of Swinbank's (1951) averaging calculus:

$$\frac{\partial}{\partial t}(\overline{\rho \varphi_k}) + \nabla \cdot (\overline{\rho \mathbf{v}}\,\overline{\varphi_k} + \overline{(\rho \mathbf{v})' \varphi_k'} + \overline{\mathbf{J}}_k) = \overline{\sigma_k}. \tag{4.9}$$

Here, σ_k represents the corresponding sources and sinks owing to phase transition processes and chemical reactions, where σ_a is always assumed being equal to zero, i.e., as mentioned before, dry air is considered as chemically inert. The term $\overline{\rho \mathbf{v} \varphi_k}$ is called the convective transport, and the quantities $\overline{(\rho \mathbf{v})' \varphi_k'}$ and $\overline{\mathbf{J}}_k$ are called the non-convective transports. Obviously, Eq. (4.9) obeys the equation of continuity (4.2), i.e.,

$$\sum_k \varphi_k = 1 \tag{4.10}$$

$$\sum_k \overline{(\rho \mathbf{v})' \varphi_k'} = \sum_k \overline{(\rho \mathbf{v})' \varphi_k} = \overline{(\rho \mathbf{v})' \sum_k \varphi_k} = \overline{(\rho \mathbf{v})'} = 0 \tag{4.11}$$

$$\sum_k \overline{\mathbf{J}}_k = 0 \tag{4.12}$$

and

$$\sum_k \overline{\sigma_k} = 0 \tag{4.13}$$

For the purpose of simplification, phase transition processes and chemical reactions are ignored in this section so that σ_v and σ_c are considered as equal to zero. The flux quantity $\overline{\mathbf{J}}_k$ may represent molecular, phoretic and sedimentation effects. As in the fully turbulent region of the ASL the flux quantity $|\overline{\mathbf{J}}_k|$ is small in comparison with the magnitude of the turbulent flux $|\overline{(\rho \mathbf{v})' \varphi_k'}|$, the former is neglected in that region. In a very thin layer adjacent to the earth's surface, however, $|\overline{\mathbf{J}}_k|$ becomes much larger than $|\overline{(\rho \mathbf{v})' \varphi_k'}|$ and, hence, the latter is negligible. Thus, $\overline{\mathbf{J}}_k$ is serving as the boundary condition at the earth's surface. Since $\rho_c \ll \rho_a + \rho_v$ (e.g., Webb et al. 1980), again we may write: $\rho = \rho_a + \rho_v + \rho_c \cong \rho_a + \rho_v$. All simplifications listed here are in complete agreement with those of Webb et al. (1980) later adopted by many other authors, i.e., if the following results differ from those of these authors, the reasons for this difference cannot be inferred from different simplifications.

As steady-state conditions must prevail for deriving eddy fluxes of momentum, sensible heat, water vapour, and trace species, we may assume that such conditions exist during the measuring interval. In doing so, Eq. (4.9) becomes

$$\nabla \cdot (\overline{\rho \mathbf{v}}\, \overline{\varphi_k} + \overline{(\rho \mathbf{v})' \varphi_k'} + \overline{\mathbf{J}_k}) = 0. \tag{4.14}$$

As $\overline{\rho_a \mathbf{v}} = \overline{\rho \mathbf{v} \varphi_a} = \overline{\rho \mathbf{v}}\,\overline{\varphi_a} + \overline{(\rho \mathbf{v})' \varphi_a'}$, Eq. (4.14) provides for dry air

$$\nabla_H \cdot (\overline{\rho_a \mathbf{v}_H} + \overline{\mathbf{J}_{a,H}}) + \frac{\partial}{\partial z}(\overline{\rho_a w} + \overline{J_{a,z}}) = 0 \tag{4.15}$$

The subscript H denotes the horizontal part of the vector operation.

4.1 Horizontal homogeneity. Because of Eq. (2.2), the term $\partial(\overline{\rho_a w})/\partial z$ is equal to zero. If we additionally assume horizontally homogeneous distribution of dry air we can infer from Eq. (4.15) that $\partial \overline{J_{a,z}}/\partial z = 0 \Rightarrow \overline{J_{a,z}} = \text{const}$. In accord with Eqs. (3.2) and (4.12), we may write $\overline{J_{a,z}} = -\overline{J_{v,z}}$. Thus, we also have $\partial \overline{J_{v,z}}/\partial z = 0 \Rightarrow \overline{J_{v,z}} = \text{const}$.

Assuming also horizontally homogeneous distributions of water vapour and trace species, Eq. (4.14) provides for water vapour ($\varphi_v = q$)

$$\frac{\partial}{\partial z}(\overline{\rho w}\,\overline{q} + \overline{(\rho w)' q'} + \overline{J_{v,z}}) = \frac{\partial}{\partial z}(\overline{\rho w}\,\overline{q} + \overline{(\rho w)' q'}) = 0 \tag{4.16}$$

and for trace species

$$\frac{\partial}{\partial z}(\overline{\rho w}\,\overline{\varphi_c} + \overline{(\rho w)' \varphi_c'} + \overline{J_{c,z}}) = 0 \tag{4.17}$$

Rearranging Eq. (3.3) provides

$$\overline{\rho w} = \overline{\rho_v w} = \overline{\rho w q} = \overline{\rho w}\,\overline{q} + \overline{(\rho w)' q'} \tag{4.18}$$

and, hence,

$$\overline{\rho w} = \frac{1}{1 - \overline{q}} \overline{(\rho w)' q'} \tag{4.19}$$

Introducing this equation into Eqs. (4.16) and (4.17) yields

$$\frac{\partial}{\partial z}\left(\frac{\overline{q}}{1 - \overline{q}} \overline{(\rho w)' q'} + \overline{(\rho w)' q'}\right) = \frac{\partial}{\partial z}\left(\frac{1}{1 - \overline{q}} \overline{(\rho w)' q'}\right) = 0 \tag{4.20}$$

leading to

$$\overline{\rho w} = \frac{1}{1 - \overline{q}} \overline{(\rho w)' q'} = \text{const.} \tag{4.21}$$

as well as

$$\frac{\partial}{\partial z}\left(\frac{\overline{\varphi_c}}{1 - \overline{q}} \overline{(\rho w)' q'} + \overline{(\rho w)' \varphi_c'} + \overline{J_{c,z}}\right) = 0. \tag{4.22}$$

Equation (4.21) states that a variation of the eddy flux of water vapour with height only depends on the mean specific humidity, $\overline{q}$, which is a very weak dependence because of the condition $\overline{q} \ll 1$, i.e., the eddy flux of water vapour is nearly invariant with height. From a physical perspective, a correction of the eddy flux of water vapour by itself, as suggested by Webb et al. (1980) and later adopted by many other authors, is, therefore, completely unjustified. Nevertheless, Eq. (4.21) is a consequence of the premise (2.2).

Note that (beside a typing error) Eq. (A6) of Liebethal and Foken (2003) seems to be similar to Eq. (4.21) even though their Eq. (A5) is inaccurate because first Eq. (A5) must read (see also the derivation of our Eq. (4.20))

$$Q = \overline{\rho w q} = \overline{\rho w}\,\overline{q} + \overline{(\rho w)' q'} = \frac{\overline{q}}{1-\overline{q}}\overline{(\rho w)' q'} + \overline{(\rho w)' q'} = \frac{1}{1-\overline{q}}\overline{(\rho w)' q'} = \text{const.} \tag{4.23}$$

and second there is the identity $\overline{(\rho \mathbf{v})' q'} = \overline{\rho \mathbf{v}' q'} + \overline{\mathbf{v}}\,\overline{\rho' q'}$ so that Eq. (A6) of Liebethal and Foken (2003) becomes

$$Q = \frac{1}{1-\overline{q}}\left(\overline{\rho w' q'} + \overline{w}\,\overline{\rho' q'}\right) \tag{4.24}$$

Obviously, Liebethal and Foken (2003) ignored the second term of the right-hand side of Eq. (4.24), as customarily done within the framework of the Boussinesq approximation.

The differentiation of the first term of Eq. (4.22) provides

$$\frac{\partial}{\partial z}\left(\frac{\overline{\varphi_c}}{1-\overline{q}}\overline{(\rho w)'\, q'}\right) = \overline{\varphi_c}\underbrace{\frac{\partial}{\partial z}\left(\frac{1}{1-\overline{q}}\overline{(\rho w)' q'}\right)}_{=0,\ see\ Eq.\ (4.20)} + \frac{1}{1-\overline{q}}\overline{(\rho w)' q'}\frac{\partial \overline{\varphi_c}}{\partial z} = \frac{1}{1-\overline{q}}\overline{(\rho w)' q'}\frac{\partial \overline{\varphi_c}}{\partial z}. \tag{4.25}$$

Equation (4.22) leads, therefore, to

$$\frac{\partial}{\partial z}\left(\overline{(\rho w)' \varphi_c'} + \overline{J_{c,z}}\right) = -\frac{1}{1-\overline{q}}\overline{(\rho w)' q'}\frac{\partial \overline{\varphi_c}}{\partial z}. \tag{4.26}$$

Apparently, the right-hand side of Eq. (4.26) does not vanish, i.e., the nearly height-invariant eddy flux of water vapour along with the vertical distribution of the mean concentration are responsible for the fact that the eddy flux of a trace species depends on height.

Using Hesselberg's (1926) density-weighted averaging procedure, as done by Kramm et al. (1995a), provides

$$\frac{1}{1-\hat{q}}\overline{\rho w'' q''} = \text{const.} \tag{4.27}$$

and

$$\frac{\partial}{\partial z}\left(\overline{\rho w'' \varphi_c''} + \overline{J_{c,z}}\right) = -\frac{1}{1-\hat{q}}\overline{\rho w'' q''}\frac{\partial \widehat{\varphi_c}}{\partial z} \tag{4.28}$$

Within the framework of the prerequisites mentioned above (these are in complete agreement with those of Webb et al. 1980, later adopted by many other authors), Eqs. (4.21), (4.26) to (4.28) can be considered as exact.

Obviously, the physical meaning of Eqs. (4.21) and (4.26) on the one hand and Eqs. (4.27) and (4.28) on the other hand is the same. This is in complete contradiction to the statement of Liebethal and Foken (2003) regarding the use of different averaging procedures with which these authors tried to explain the large difference between their flux correction estimates and those of Kramm et al.

(1995a), illustrated here in Figure 1. These large differences can only be explained by the fact that the conventional Webb correction is based on elements of the Boussinesq approximation so that the corresponding correction equations are not in agreement with the governing principles of conservation laws.

To estimate the flux correction effect on the basis of the computed mean vertical velocity, we define the mean transfer velocity, $v_{e,k}$ (either the deposition or the exhalation velocity, see, e.g., Kramm et al. 1995b), of trace species given by

$$v_{e,k} = -\frac{F_c}{\overline{\rho}\widehat{\varphi_k}} = -\frac{\overline{\rho w'' \varphi_k''}}{\overline{\rho}\widehat{\varphi_k}}. \tag{4.29}$$

In accord with Eqs. (2.1) and (4.29), we may write

$$F_c = \overline{w\rho_k} = \overline{\rho w \varphi_k} = \overline{\rho}\hat{w}\widehat{\varphi_k} + \overline{\rho w'' \varphi_k''} = \overline{\rho}\hat{w}\widehat{\varphi_k} - \overline{\rho}\widehat{\varphi_k} v_{e,k} = -\overline{\rho}\widehat{\varphi_k} v_{e,k}\left(1 - \frac{\hat{w}}{v_{t,k}}\right) \tag{4.30}$$

where the ratio

$$C = \frac{\hat{w}}{v_{e,k}} \tag{4.31}$$

defines the flux correction. To derive the mean vertical velocity, we rearrange Eq. (2.2) to

$$0 = \overline{\rho_a w} = \overline{\rho w} - \overline{\rho_v w} = \overline{\rho}\hat{w} - \overline{\rho w q} = \overline{\rho}\hat{w} - \overline{\rho}\hat{w}\hat{q} - \overline{\rho w'' q''} = \overline{\rho}\hat{w}(1-\hat{q}) - \overline{\rho\, w'' q''} \tag{4.32}$$

which leads to

$$\hat{w} = \frac{\overline{\rho w'' q''}}{\overline{\rho}(1-\hat{q})}. \tag{4.33}$$

If we adopt Eq. (2.2), the most prominent prerequisite on which the conventional Webb correction is based, as correct, Eq. (4.33) will be exact. Obviously, it is in contradiction to Eq. (2.10) because it only contains the eddy flux of water vapour, but not the heat flux term $\overline{w'T'}$. We can infer from Figure 2 that the flux correction is considerably smaller when only the vertical eddy flux of water vapour is considered. Since the variation of the mean air density across the ASL is very small, and the term $\overline{\rho w'' q''}/(1-\hat{q})$ is invariant with height (see Eq. (4.27)), the mean vertical velocity given by Eq. (4.33) only varies hardly, i.e., the computed mean vertical velocity at a certain level within the ASL is very close to zero because at the rigid surface of the earth not only the instantaneous value of the vertical velocity vanishes there, but also its mean value and its fluctuating one.

Taking the Bowen ratio

$$\beta = \frac{H}{E} = \frac{c_p \overline{\rho w'' \Theta''}}{L_v \overline{\rho w'' q''}}, \tag{4.34}$$

into account, as done in the case of Eq. (2.12), the mean vertical velocity can be expressed as a function of the sensible heat flux, H, and the Bowen ratio, β, by (see also Kramm et al. 1995a)

$$\hat{w} = \frac{H}{\overline{\rho}(1-\hat{q})\beta L_v}. \tag{4.35}$$

Note that the equation of state for moist air was not used, neither in its complete form (see Appendix A) nor in an approximated one, as done by Webb et al. (1980). Using Eq. (4.35) the flux correction can be expressed by

$$C = \frac{H}{\overline{\rho}(1-\hat{q})\beta L_v\, v_{e,k}} \cong \frac{H}{\overline{\rho}\beta L_v v_{e,k}} \tag{4.36}$$

because $\hat{q} \ll 1$. This formula may be used to estimate the flux correction of any trace species.

Integrating Eq. (4.26) over the layer $0 \leq z \leq z_R$ of the ASL, where z_R is the reference height at which, for instance, the eddy covariance measurements are performed, yields

$$F_{c,R} = \overline{(\rho w)'\varphi_c'}\big|_{z=z_R} = F_{c,S} - \frac{\overline{\varphi_{c,R}} - \overline{\varphi_{c,S}}}{1-\overline{q_R}}\overline{(\rho w)'q'}\big|_{z=z_R}. \tag{4.37}$$

Here, $F_{c,S} = \overline{J_{c,z}}\big|_{z=0}$ is the flux of a trace species at the earth's surface ($z = 0$) i.e., the dry deposition or exhalation flux, $\overline{\varphi_{c,S}}$ is the corresponding mean mass fraction, $F_{c,R}$ is the eddy flux of this trace species at z_R, and $\overline{\varphi_{c,R}}$ and $\overline{q_R}$ are the mean mass fraction and the mean specific humidity at the same height, respectively. Since the term $Q = \overline{(\rho w)'q'}/(1-\overline{q})$ is invariant with height (see Eq. (4.21)), it can also be taken at the height z_R. Combining Eqs. (4.19), (4.21) and (4.37) with each other yields then

$$F_{c,R} = \overline{(\rho w)'\varphi_c'}\,\big|_{z=z_R} = F_{c,S} - (\overline{\varphi_{c,R}} - \overline{\varphi_{c,S}})\,\overline{\rho w} = F_{c,S} - (\overline{\varphi_{c,R}} - \overline{\varphi_{c,S}})\,Q. \tag{4.38}$$

This equation illustrates that the relative importance of the flux correction is dependent on both the evaporation and the difference of the mass fractions, $\Delta\overline{\varphi_c} = \overline{\varphi_{c,R}} - \overline{\varphi_{c,S}}$. Using Hesselberg's (1926) density-weighted averaging procedure provides (Kramm et al. 1995a)

$$F_{c,R} = \overline{\rho w''\varphi_c''}\big|_{z=z_R} = F_{c,S} - (\widehat{\varphi_{c,R}} - \widehat{\varphi_{c,S}})\overline{\rho}\hat{w} = F_{c,S} - (\widehat{\varphi_{c,R}} - \widehat{\varphi_{c,S}})Q, \tag{4.39}$$

where now, in accord with Eq. (4.33), the quantity Q is given by $Q = \overline{\rho w''q''}/(1-\hat{q})$. Therefore, one may conclude that Eq. (4.38) substantially agrees with Eq. (4.39). Equations (4.38) and (4.39) may also serve as an extrapolation prescription to relate the flux term $F_{c,R}$ to that at the earth's surface, $F_{c,S}$.

4.2 Advective effects. As already shown by Paw U et al. (2000), when, in addition, advective effects are considered, such correction equations become more complex. By using, for instance, Hesselberg's density-weighted averaging procedure the balance equations for dry air, water vapour, and trace constituents derived for a turbulent system reads (Kramm et al. 1995a)

$$\frac{\partial}{\partial t}(\overline{\rho}\widehat{\varphi_k}) + \nabla\cdot(\overline{\rho}\hat{v}\widehat{\varphi_k} + \overline{\rho v''\varphi_k''} + \overline{J_k}) = \overline{\sigma_k}. \tag{4.40}$$

From a physical point of view, this equation substantially agreement with Eq. (4.9). Assuming again that steady-state conditions exist during the measuring interval, Eq. (4.40) leads to

$$\nabla\cdot(\overline{\rho}\hat{\mathbf{v}}\widehat{\varphi_k} + \overline{\rho\mathbf{v}''\varphi_k''} + \overline{\mathbf{J}}_k) = \overline{\sigma_k}. \tag{4.41}$$

In the case of dry air we again assume that σ_a is equal to zero. Introducing Eq. (2.2) into Eq. (4.15) yields then

$$\nabla_H \cdot (\overline{\rho \mathbf{v}_H \varphi_a} + \overline{\mathbf{J}_{a,H}}) + \frac{\partial}{\partial z}(\overline{J_{a,z}}) = 0. \tag{4.42}$$

Again, the subscript H denotes the horizontal part of the vector operation. The term $\overline{\rho \mathbf{v}_H \varphi_a}$ can be rearranged in the following manner:

$$\overline{\rho\, \mathbf{v}_H\, \varphi_a} = \overline{\rho \mathbf{v}_H (1-q)} = \overline{\rho \mathbf{v}_H} - \overline{\rho \mathbf{v}_H q} = \overline{\rho}\widehat{\mathbf{v}_H}(1-\hat{q}) - \overline{\rho \mathbf{v}_H'' q''} \tag{4.43}$$

Equation (4.42) reads then

$$\nabla_H \cdot (\overline{\rho}\widehat{\mathbf{v}_H}(1-\hat{q}) - \overline{\rho \mathbf{v}_H'' q''} + \overline{\mathbf{J}_{a,H}}) + \frac{\partial}{\partial z}\overline{J_{a,z}} = 0 \tag{4.44}$$

Since $\overline{J_a} + \overline{J_v} = 0$ (see Eqs. (3.2) and (4.12)), we may write

$$\nabla_H \cdot (\overline{\rho}\widehat{\mathbf{v}}_H(1-\hat{q}) - \overline{\rho \mathbf{v}_H'' q''} - \overline{\mathbf{J}_{v,H}}) - \frac{\partial}{\partial z}\overline{J_{v,z}} = 0. \tag{4.45}$$

For water vapour we obtain ($\varphi_V = q$)

$$\nabla \cdot (\overline{\rho}\hat{\mathbf{v}}\hat{q} + \overline{\rho \mathbf{v}'' q''} + \overline{\mathbf{J}_v}) = \nabla_H \cdot (\overline{\rho}\hat{\mathbf{v}}_H\hat{q} + \overline{\rho \mathbf{v}_H'' q''} + \overline{\mathbf{J}_{v,H}}) + \frac{\partial}{\partial z}(\overline{\rho}\hat{w}\hat{q} + \overline{\rho w'' q''} + \overline{J_{v,z}}) = 0 \tag{4.46}$$

or

$$\nabla_H \cdot (\overline{\rho}\hat{\mathbf{v}}_H\hat{q} + \overline{\rho \mathbf{v}_H'' q''} + \overline{\mathbf{J}_{v,H}}) = -\frac{\partial}{\partial z}(\overline{\rho}\hat{w}\hat{q} + \overline{\rho w'' q''} + \overline{J_{v,z}}) \tag{4.47}$$

Again, σ_v is assumed to be zero, i.e., phase transition processes are ignored. Introducing this equation into Eq. (4.45) provides

$$\nabla_H \cdot (\overline{\rho}\hat{\mathbf{v}}_H) = -\frac{\partial}{\partial z}(\overline{\rho}\hat{w}\hat{q} + \overline{\rho w'' q''}). \tag{4.48}$$

Note that the flux terms $\overline{\mathbf{J}_{v,H}}$ and $\overline{J_{v,z}}$ have been eliminated from that equation and must not further occur. By considering steady-state conditions, we can infer from the equation of continuity (4.5)

$$\nabla_H \cdot (\overline{\rho}\widehat{\mathbf{v}}_H) = -\frac{\partial}{\partial z}(\overline{\rho}\hat{w}). \tag{4.49}$$

Note that this equation reflects the so-called anelastic approximation usually considered in non-hydrostatic numerical models of the atmosphere to eliminate sound waves as a possible solution (e.g., Ogura and Phillips 1962, Pielke 1984). Dutton and Fichtl (1969) called this relation the equation of continuity for deep convection. Combining Eqs. (4.48) and (4.49) yields then

$$\frac{\partial}{\partial z}(\overline{\rho}\hat{w}) = \frac{\partial}{\partial z}(\overline{\rho}\hat{w}\hat{q} + \overline{\rho w'' q''}) \tag{4.50}$$

or

$$\frac{\partial}{\partial z}(\overline{\rho}\hat{w}(1-\hat{q})) = \frac{\partial}{\partial z}(\overline{\rho w'' q''}). \tag{4.51}$$

In the following even terrain is generally assumed for the purpose of simplification (for irregular terrain that may also cause a vertical wind velocity, see Kramm et al. 2004). This, of course, does not include horizontally homogeneous distribution of wind speed, water vapour or trace constituents. Integrating Eq. 4.51) over the layer $0 \leq z \leq z_R$ yields then

$$\widehat{w}_R = \frac{\overline{\rho w'' q''}\big|_{z=z_R}}{\overline{\rho_R}(1-\widehat{q}_R)} \tag{4.52}$$

because $\hat{w}$ and $\overline{\rho\, w'' q''}$ vanish at the rigid surface of the earth. Again, the subscript R characterizes the values at the reference height z_R. It seems that Eq. (4.52) completely agrees with Eq. (4.33), but only in the latter the term $Q = \overline{\rho w'' q''}/(1-\hat{q})$ is invariant with height. Again, there is no indication that any heat flux term must occur.

For trace gases we can infer from Eq. (4.41)

$$\nabla_H \cdot (\overline{\rho}\widehat{\mathbf{v}}_H\widehat{\varphi}_c + \overline{\rho \mathbf{v}_H'' \varphi_c''} + \overline{\mathbf{J}_{c,H}}) + \frac{\partial}{\partial z}(\overline{\rho}\hat{w}\widehat{\varphi}_c + \overline{\rho w'' \varphi_c''} + \overline{J_{c,z}}) = \overline{\sigma_c} \tag{4.53}$$

Integrating Eq. (4.53) over the layer $0 \leq z \leq z_R$ provides

$$\left\{\overline{\rho}\hat{w}\widehat{\varphi}_c + \overline{\rho w'' \varphi_c''} + \overline{J_{c,z}}\right\}_{z=z_R} - F_{c,S} = \int_0^{z_R} \overline{\sigma_c} dz - \int_0^{z_R} \nabla_H \cdot \left(\overline{\rho}\hat{\mathbf{v}}_H\hat{\varphi}_c + \overline{\rho \mathbf{v}_H'' \varphi_c''} + \overline{\mathbf{J}_{c,H}}\right) dz \tag{4.54}$$

or by considering Eq. (4.52)

$$\begin{aligned}\left\{\overline{\rho w'' \varphi_c''} + \overline{J_{c,z}}\right\}_{z=z_R} = F_{c,S} - \frac{\hat{\varphi}_{c,R}}{1-\hat{q}_R}\overline{\rho w'' q''}\big|_{z=z_R} + \int_0^{z_R} \overline{\sigma_c} dz \\ - \int_0^{z_R} \nabla_H \cdot (\overline{\rho}\hat{\mathbf{v}}_H\hat{\varphi}_c + \overline{\rho \mathbf{v}_H'' \varphi_c''} + \overline{\mathbf{J}_{c,H}}) dz.\end{aligned} \tag{4.55}$$

Here, $F_{c,S} = \overline{J_{c,z}}\big|_{z=z_R}$ is, again, the trace gas flux at the earth's surface. In contrast to Eq. (4.39), this equation also contains advective effects. Furthermore, only the mass fraction $\hat{\varphi}_{c,R}$ at the reference height occurs, but not $\Delta\hat{\varphi}_c = \hat{\varphi}_{c,R} - \hat{\varphi}_{c,S}$. This difference can be attributed to the term $Q = \overline{\rho w'' q''}/(1-\hat{q})$ that is invariant with height when water vapour is horizontally homogeneously distributed. Nevertheless, both equations substantiate that an eddy heat flux term must not occur.

The vertical average of any height-dependent quantity $\xi(z)$ is defined by (e.g., Fortak 1969, Lange 1985, Stull 1988, Garratt 1994)

$$\langle \xi \rangle = \frac{1}{z_R}\int_0^{z_R} \xi(z) dz. \tag{4.56}$$

If we consider the reference height as fixed, i.e., $\nabla_H z_R = 0$, we may write (see also Kramm et al. 2004)

$$\left.\begin{aligned}\int_0^{z_R} \nabla_H \cdot (\overline{\rho}\hat{\mathbf{v}}_H\hat{\varphi}_c + \overline{\rho \mathbf{v}_H'' \varphi_c''} + \overline{\mathbf{J}_{c,H}}) dz &= \nabla_H \cdot \int_0^{z_R} (\overline{\rho}\hat{\mathbf{v}}_H\hat{\varphi}_c + \overline{\rho \mathbf{v}_H'' \varphi_c''} + \overline{\mathbf{J}_{c,H}}) dz \\ &= z_R \nabla_H \cdot (\langle \overline{\rho}\hat{\mathbf{v}}_H\hat{\varphi}_c \rangle + \langle \overline{\rho \mathbf{v}_H'' \varphi_c''} \rangle + \langle \overline{\mathbf{J}_{c,H}} \rangle)\end{aligned}\right\} \tag{4.57}$$

Thus, Eq. (4.55) can be rearranged as following:

$$\left.\begin{aligned}\{\overline{\rho w'' \varphi_c''} + \overline{J_{c,z}}\}_{z=z_R} = F_{c,S} - \frac{\hat{\varphi}_{c,R}}{1-\hat{q}_R}\overline{\rho w'' q''}\big|_{z=z_R} \\ + z_R\{\langle\overline{\sigma_c}\rangle - \nabla_H \cdot (\langle\overline{\rho}\hat{\mathbf{v}}_H\hat{\varphi}_c\rangle + \langle\overline{\rho \mathbf{v}_H'' \varphi_c''}\rangle + \langle\overline{\mathbf{J}_{c,H}}\rangle)\}\end{aligned}\right\} \tag{4.58}$$

This equation can be assessed as exact when steady-state conditions are fulfilled.

5. Approximate equations. Since the Hesselberg average can be related to that of Reynolds by (e.g., van Mieghem 1973, Cox 1995, Kramm et al. 1995a, Herbert 1995)

$$\hat{\chi} = \overline{\chi} + \frac{\overline{\rho'\chi'}}{\overline{\rho}} = \overline{\chi}\left\{1 + \frac{\overline{\rho'\chi'}}{\overline{\rho}\,\overline{\chi}}\right\}, \tag{5.1}$$

where $\hat{\chi}$ and $\overline{\chi}$ are nearly equal when $\overline{\rho'\chi'}/\{\overline{\rho}\,\overline{\chi}\} \ll 1$ (as customarily assumed within the framework of the Boussinesq approximation), the equations finally derived in Chapter 4 may be simplified for more practical purposes. Such simplified equations are presented in the following:

5.1 Horizontal homogeneity. According to Eq. (5.1), the simplification of Eqs. (4.27) and (4.28) leads to

$$\frac{1}{1-\hat{q}}\overline{\rho w'' q''} \cong \frac{1}{1-\overline{q}}\overline{\rho}\,\overline{w'q'} = \text{const.} \tag{5.2}$$

and

$$\frac{1}{1-\hat{q}}\overline{\rho w'' q''}\frac{\partial\hat{\varphi}_c}{\partial z} \cong \frac{1}{1-\overline{q}}\overline{\rho}\,\overline{w'q'}\frac{\partial\overline{\varphi_c}}{\partial z}. \tag{5.3}$$

These equations should be used when pressure fluctuation measurements are not available so that neither the pressure transport term $\overline{w'\,p'}$ is available for flux correction (see Eqs. (2.19) and (3.8)) nor density-weighted averages can be realised in practise. Note that the term on the right-hand side of Eq. (5.2) was denoted by Webb et al. (1980) as eddy flux of water vapour (see their Eq. (23)). In accord with equations (5.1) and (5.2), Eq. (4.33) might be replaced by

$$\overline{w} \cong \frac{1}{1-\overline{q}}\overline{w'q'} = \text{const.} \tag{5.4}$$

Introducing this expression into Eq. (2.1) provides finally

$$F_c = \overline{w'\rho_c'} + \frac{\overline{\rho_c}}{1-\overline{q}}\overline{w'q'} \tag{5.5}$$

It substantially agrees with Eqs. (4.38) and (4.39). Again, in contrast to the conventional Webb correction, only the eddy flux of water vapour occurs in the correction equation. Note that replacing ρ_c by ρ_v yields

$$Q = \overline{w'\rho_v'} + \frac{\overline{\rho_v}}{1-\overline{q}}\overline{w'q'} = \overline{w'\rho_v} + \frac{\overline{\rho_v}}{1-\overline{q}}\overline{w'q'} = \overline{w'\rho q} + \frac{\overline{\rho_v}}{1-\overline{q}}\overline{w'q'}, \tag{5.6}$$

where $\overline{w'\rho q}$ and $\overline{\rho_v}$ are given by $\overline{w'\rho q} = \overline{\rho}\,\overline{w'q'} + \overline{q}\,\overline{w'\rho'} + \overline{w'\rho'q'}$ and $\overline{\rho_v} = \overline{\rho}\,\overline{q} + \overline{\rho'q'}$, respectively. Introducing these expressions into Eq. (5.6) yields then

$$Q = \frac{\overline{\rho} + \overline{\rho'q'}}{1-\overline{q}}\overline{w'q'} + \overline{w'\rho'q'} \tag{5.7}$$

or

$$Q \cong \frac{1}{1-\overline{q}}\overline{\rho}\overline{w'q'} = \text{const.}, \tag{5.8}$$

the Boussinesq approximated form of Eq. (5.7).

Even though the right-hand sides of Eqs. (5.2) to (5.5) and (5.8) are also approximated in the sense of Boussinesq, this kind of simplification is much more reasonable than that described in Chapters 2 and 3 because (a) the simplification was consistently carried out on the basis of the Eqs. (4.27), (4.28), and (4.33) that were exactly and finally derived, and (b) the simplification does not change the physical meaning of these equations.

5.2 Advective effects. Usually, the non-convective horizontal transport terms $\langle\overline{\rho \mathbf{v}''_H \varphi''_c}\rangle$ and $\langle\overline{\mathbf{J}_{c,H}}\rangle$ are much smaller than the horizontal convective transport term $\langle\overline{\rho}\hat{\mathbf{v}}_H\hat{\varphi}_c\rangle$. As mentioned before, in the fully turbulent region of the ASL the flux quantity $|\overline{J_{c,z}}|$ is very small in comparison with $|\overline{\rho w''\varphi''_c}|$. Thus, Eq. (4.58) may be approximated by

$$F_{c,R} = \overline{\rho w''\varphi''_c}\big|_{z=z_R} = F_{c,S} - \frac{\hat{\varphi}_{c,R}}{1-\hat{q}_R}\overline{\rho w''q''}\big|_{z=z_R} + z_R\{\langle\overline{\sigma_c}\rangle - \nabla_H\cdot\langle\overline{\rho}\hat{\mathbf{v}}_H\hat{\varphi}_c\rangle\}. \tag{5.9}$$

If, in addition, chemical reactions are negligible, we have

$$F_{c,R} = \overline{\rho w''\varphi''_c}\big|_{z=z_R} = F_{c,S} - \frac{\hat{\varphi}_{c,R}}{1-\hat{q}_R}\overline{\rho w''q''}\big|_{z=z_R} - z_R\nabla_H\cdot\langle\overline{\rho}\hat{\mathbf{v}}_H\hat{\varphi}_c\rangle \tag{5.10}$$

This equation reflects the correction owing to both the vertical eddy flux of water vapour and the horizontal advection. Replacing the density-weighted averages by the conventional Reynolds averages, as done in section 5.1, leads then to

$$F_{c,R} = \overline{\rho}\overline{w'\varphi'_c}\big|_{z=z_R} = F_{c,S} - \frac{\overline{\varphi_{c,R}}}{1-\overline{q_R}}\overline{\rho}\overline{w'q'}\big|_{z=z_R} + z_R\{\langle\overline{\sigma_c}\rangle - \nabla_H\cdot\langle\overline{\rho}\,\overline{v_H}\,\overline{\varphi_c}\rangle\} \tag{5.11}$$

and

$$F_{c,R} = \overline{\rho}\overline{w'\varphi'_c}\big|_{z=z_R} = F_{c,S} - \frac{\overline{\varphi_{c,R}}}{1-\overline{q_R}}\overline{\rho}\overline{w'q'}\big|_{z=z_R} - z_R\nabla_H\cdot\langle\overline{\rho}\,\overline{v_H}\,\overline{\varphi_c}\rangle. \tag{5.12}$$

The latter two equations may be considered for practical purposes. They may also serve as an extrapolation prescription to relate the flux $F_{c,R}$ to that at the earth's surface, $F_{c,S}$, as required in determining either the *true* deposition flux or the *true* emission flux.

6. Final remarks and conclusions. The correction of eddy covariance terms customarily called the Webb correction was presented and assessed. It was shown that the conventional Webb correction is based on elements of a Boussinesq approximation. Such elements, however, should not be considered while any kind of flux correction equation is derived because flux correction equations that are, completely or partly, Boussinesq approximated violate conservation laws like the equation of continuity and the balance equations for water vapour and trace species derived for turbulent systems. Furthermore, it was shown that density-weighted averaging procedures like those of Hesselberg (1926) and Swinbank (1951) that serve to completely satisfy all requirements associated with such conservation laws provide physically similar correction equations. If the most prominent prerequisite of the conventional Webb correction, namely that no vertical flux of dry air exists, the correction equations derived on the basis of these density-weighted averaging procedures can be considered as exact. In contrast to the conventional Webb correction, (a) these correction equations are not affected by the eddy flux of sensible heat and (b) the water vapour flux is invariant with height. Consequently, the correction effects based on such density-weighted averaging procedures are much smaller than those derived by the conventional Webb correction approach. Using density-weighted averages in practise, however, requires the measurements of pressure fluctuations. The same is true when the pressure transport term $\overline{w'p'}$ is considered. This term is usually ignored within the framework of the conventional Webb correction.

Flux correction equations for more practical purposes were derived on the basis of the exact correction equations by a self-consistent Boussinesq approximation. It was shown that this kind of the simplification does not change the physical meaning of the exact correction equations.

It has to be recognized that for the budget of a trace species the flux $F_{c,S}$, which contributes to the loss or gain of a trace species by dry deposition or exhalation, is relevant rather than $F_{c,R}$. In studies on the budget of a trace species, the latter mainly serves to estimate $F_{c,S}$, where micrometeorological principles are considered to infer the dry deposition or exhalation flux of a trace species from the turbulent flux $F_{c,R}$. The development of such an extrapolation prescription can only be carried out when full attention is paid to the governing conservation laws such as equation of continuity and the balance equations of water vapour and trace species. It is obvious that this aspect plays no role in most of the papers dealing with the conventional Webb correction.

Appendix A: Derivation of Equations (3.7) and (3.9). The equation of state for moist air reads

$$p = \rho R_a \, T_v \tag{A.1}$$

All symbols have the same meaning as before. The virtual temperature is defined by

$$T_v = T\left\{1 + \left(\frac{R_v}{R_a} - 1\right) q\right\} \cong T(1 + 0.608q), \tag{A.2}$$

where R_v is the gas constant of water vapour, and q is, again, the specific humidity. Averaging Eq. (A1) in the sense of Reynolds yields

$$\overline{p} = R_a(\overline{\rho}\,\overline{T_v} + \overline{\rho' T_v'}) \tag{A.3}$$

If one expresses the instantaneous quantities in Eq. (A1) by their mean values and the deviations from those (i.e., Reynolds decomposition), one will obtain

$$\overline{p} + p' = R_a(\overline{\rho} + \rho')(\overline{T_v} + T_v') = R_a(\overline{\rho}\,\overline{T_v} + \rho'\overline{T_v} + \overline{\rho}T_v' + \rho' T_v'). \tag{A.4}$$

Combining Eqs. (A3) and (A4) yields then

$$p' = R_a(\rho'\overline{T_v} + \overline{\rho}T_v' + \rho' T_v' - \overline{\rho' T_v'}). \tag{A.5}$$

If we divide this equation by the mean pressure given by Eq. (A3), we will obtain

$$\frac{p'}{\overline{p}} = \frac{\rho'\overline{T_v} + \overline{\rho}T_v' + \rho' T_v' - \overline{\rho' T_v'}}{\overline{\rho}\,\overline{T_v} + \overline{\rho' T_v'}} \tag{A.6}$$

or

$$\frac{p'}{\overline{p}} = \frac{\rho'}{\overline{\rho}}\frac{1}{1+\frac{\overline{\rho' T_v'}}{\overline{T_v}}} + \frac{T_v'}{\overline{T_v}}\frac{1}{1+\frac{\overline{\rho' T_v'}}{\overline{\rho}}} + \frac{\rho' T_v' - \overline{\rho' T_v'}}{\overline{\rho}\overline{T_v}\left(1+\frac{\overline{\rho' T_v'}}{\overline{\rho}\overline{T_v}}\right)}. \tag{A.7}$$

If this equation is Boussinesq approximated, i.e., all second moments containing air density fluctuations are ignored except in those terms expressing the effect of the gravity field $-\nabla\phi$ on the density fluctuations ρ' occurring in the mass field ρ (e.g., van Mieghem 1973, Holton 1979), and $\rho' T_v' \cong \overline{\rho' T_v'}$ is assumed (usually not true), we will obtain the classical simplification (e.g., Holton 1979, Businger 1982, Stull 1988)

$$\frac{p'}{\overline{p}} \cong \frac{\rho'}{\overline{\rho}} + \frac{T_v'}{\overline{T_v}}. \tag{A.8}$$

Ignoring the pressure fluctuations yields finally

$$\frac{\rho'}{\overline{\rho}} \cong -\frac{T_v'}{\overline{T_v}} = -\frac{T'(1+0.608\overline{q}) + 0.608(\overline{T}q' + T'q' - \overline{T'q'})}{\overline{T}(1+0.608\overline{q}) + 0.608\overline{T'q'}} \tag{A.9}$$

or(when all terms containing the specific humidity are neglected)

$$\frac{\rho'}{\overline{\rho}} \cong -\frac{T'}{\overline{T}}. \tag{A.10}$$

Obviously, Eqs. (A9) and (A10) are results of the Boussinesq approximation, where, in addition, pressure fluctuations are ignored.

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