Thermal properties + heat transfer

Daniel Pringle, Fall 2006, GEOS 692, University Alaska Fairbanks

Heat capacity

Thermal conductivity

Variability
- Rock type
- Anisotropy
- Pressure
- Temperature

Mineralology
Fabric/structure
Conductivity mechanism
Applications of Thermophysical Properties

Geothermal heat flow:
   Earth’s age and structure

Rock genesis processes:
   rates of cooling, heat transfer

Reconstruction of surface temperature history
   borehole temperature profiles + climate reconstruction

Engineering and geo-technical applications:
   bore hole heat dissipation
   permafrost degradation
   stability of buried gas clathrates
Main thermo-physical properties

 Specific Heat \( c \) [J kg\(^{-1}\)K\(^{-1}\)]
Heat required to raise the temperature of unit mass of material by 1 degree.
\( c \rho = c_{\text{vol}} \) “volumetric heat capacity” \( \approx 2 \) MJ kg\(^{-1}\)K\(^{-1}\)
\( \rightarrow \) Bulk property, scalar

 Thermal Conductivity \( k \) or \( \lambda \) [W m\(^{-1}\)K\(^{-1}\)]
Heat flow per unit temperature gradient.
\( \rightarrow \) Transport property, tensor

 Thermal Diffusivity \( D \) or \( K \) or \( \alpha \) [m\(^2\)/s]
Governs rate of spread of temperature disturbances.

Latent heat, coefficient of thermal expansion.
Specific heat

Usage:

\[ Q = mc\Delta T \]

Definition:

\[ c_p = \left( \frac{\partial E}{\partial T} \right)_p \quad c_v = \left( \frac{\partial E}{\partial T} \right)_v \]

Specific heat constant pressure, constant volume

- Temperature ↔ kinetic energy within material
- energy modes in electrically insulating crystals: lattice vibrations
- lattice vibrations are quantized: phonons

"Collective excitations"
"Quanta" of lattice vibrations
Sound version of photons
Phonons and specific heat

- statistics of phonons: Einstein and Debye theories: $E(T) \rightarrow c(T)$

$E = hf = \frac{h\nu}{\lambda}$

$\theta_D$ ‘Debye temperature’ for each material

$\theta_D(Si) \sim 750$ K, $\theta_D(Ge) \sim 500$ K,
Heat capacity of composites: easy!

mass-weighted averages

\[ C_{vol,\text{eff}} = v_1 c_1 \rho_1 + v_2 c_2 \rho_2 + \ldots \]

\( v_i \) – volume fractions, \( \rho_i \), densities

<table>
<thead>
<tr>
<th></th>
<th>Specific heat, ( c ) [J kg(^{-1})K(^{-1})] ( ~ 300 K)</th>
<th>( C_\rho ) [MJ m(^{-3})K(^{-1})]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air</td>
<td>1000</td>
<td>1 x 10(^{-3})</td>
</tr>
<tr>
<td>Water</td>
<td>4184</td>
<td>4</td>
</tr>
<tr>
<td>Ice</td>
<td>2100 (0 ºC)</td>
<td>2</td>
</tr>
<tr>
<td>Quartz</td>
<td>733</td>
<td>2</td>
</tr>
<tr>
<td>Rock minerals</td>
<td>700-800</td>
<td>( \sim 2.3 \pm 0.5)</td>
</tr>
</tbody>
</table>

Roy, Clauser & Huenges
Sea ice heat capacity

- S = 12.5 ppt
- S = 8.5 ppt
- S = 6 ppt

Thermal Conductivity of Rocks

Variability
- Rock type
- Anisotropy
- Pressure
- Temperature

Mineralogy
Fabric/structure
Conductivity mechanism
Conductivity Variability

ROCK

thermal conductivity in W/mK

Granite, Syenite
Diorite, Gabbro
Dunite, Pyroxenite
Basalt
Porphyry
Gneiss, Schist
Quartzite
Marble
Sandstone
Shale, Siltstone
Limestone
Dolomite
Anhydrite
Sand
Loam

Main influences

gas saturated ← porous
← water saturated
→ dense

Schön
Rock type Analysis - Clauser & Huenges

Rock Type

- Sedimentary
- Volcanic
- Plutonic
- Metamorphic

Paramount Factors

- porosity, anisotropy
- porosity
- dominant mineral phase
- anisotropy, dominant mineral phase

Metamorphic rocks; high/low quartz content

Volcanic rocks; high/low porosity

Clauser & Huenges
Porosity

Hawaiian basalts

\[ k(\text{water}) \sim 0.6 \text{ Wm}^{-1}\text{K}^{-1} \]

\[ k(\text{air}) \sim 0.02 \text{ Wm}^{-1}\text{K}^{-1} \]

Clauser & Huenges
Temperature

\[ k \sim T^{-1} \]

Quartzite
Quartz-poor (incl. marble, slate albitite, amphibolite)

Metamorphic
Basalt glass, obsidian, diabase dolerite

Volcanic
Basalt, rhyolite, dacite, tuff

Many empirical fits for \( k(T) \): local areas, different rock types (see Schön, Clauser & Huenges)

Clauser & Huenges
Heat transfer

What is being transported, and how?

Thermal (kinetic) energy transferred: conduction, radiation, convection.

Radiative ‘conductivity’: \( k_r \propto \frac{T^3}{\alpha_{opt}} \)

\( T \) – temperature [K]
\( \alpha \) – optical absorption coeff.

Clauser & Huenges
Phonons again

Electrical insulators as ‘phonon gas’

Gas theory result applies!

\[ k = \frac{1}{3} c u \ell \]

c, specific heat
u, phonon velocity
\ell, mean free path

~constant
distance between scattering events

Kittel
Phonon mean free path

\[ \ell \rightarrow \text{crystal size} \]
\[ \ell \sim \text{constant}, \]
\[ c \sim T^3 \rightarrow k \sim T^3 \]

phonon density increases as \( N \sim T \)

\[ \ell \sim T^{-1} \rightarrow k \sim T^{-1} \]
Phonon mean free path

\[ l \rightarrow \text{crystal size} \]
\[ l \sim \text{constant,} \]
\[ c \sim T^3 \rightarrow k \sim T^3 \]

phonon density increases as \( N \sim T \)

\[ l \sim T^{-1} \rightarrow k \sim T^{-1} \]

\( T > 300 \text{ K}: \) crystalline solids: \( k \sim T^{-1}, \)
small crystallites and amorphous solids (eg. glasses) \( k \sim c \)

Kittel
Quartz Conductivity T-dependence

Ordered: 
\( \ell \) limited by phonon density

Disordered: 
mfp limited scale of disorder

fused quartz \( \ell \sim 8 \text{ Å} \)
Si\(_0\)\(_2\) tetrahedra \( \sim 7 \text{ Å} \)

Figure 6.5. Thermal conductivity of quartz.

Parrott & Stuckes, Kittel
Thermal Conductivity of Rocks

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Mineralogy

Fabric/structure

Conductivity mechanism

Phonons + radiative
Thermal conductivity of minerals

### Thermal properties of some rock forming minerals:

References: Melnikov et al. 1975 (M); Dortman 1976 (D); Cermak, Rybach 1982 (C); Quiel 1975 (Q); Bragaud et al. 1989, 1992 (B); Huenges et al. 1989 (H); The data for $\lambda$ are related to room temperatures and for $c$ to 0 °C in Ref. C

<table>
<thead>
<tr>
<th>Mineral</th>
<th>$\lambda$ in W m$^{-1}$ K$^{-1}$</th>
<th>$c$ in kJ kg$^{-1}$ K$^{-1}$</th>
<th>Ref</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hematite</td>
<td>11.2 - 14.7</td>
<td>0.61</td>
<td>C, M, D</td>
</tr>
<tr>
<td>Magnetite</td>
<td>4.7 - 5.3</td>
<td>0.6</td>
<td>C, M, D</td>
</tr>
<tr>
<td>Pyrite</td>
<td>19.2</td>
<td>0.5</td>
<td>C, H</td>
</tr>
<tr>
<td>Fluorite</td>
<td>9.5</td>
<td>0.85</td>
<td>C</td>
</tr>
<tr>
<td>Chlorite</td>
<td>4.2 - 5.14</td>
<td>0.6</td>
<td>C, M</td>
</tr>
<tr>
<td>Halite</td>
<td>5.3 - 10</td>
<td>0.79 - 0.84</td>
<td>M</td>
</tr>
<tr>
<td>Apatite</td>
<td>1.37 - 1.4</td>
<td>0.7</td>
<td>C, M</td>
</tr>
<tr>
<td>Calcite</td>
<td>3.25 - 3.9</td>
<td>0.79 - 0.83</td>
<td>C, M</td>
</tr>
<tr>
<td>Dolomite</td>
<td>3.9 - 5.5</td>
<td>0.86 - 0.93</td>
<td>C, M</td>
</tr>
<tr>
<td>Anhydrite</td>
<td>4.6 - 5.75</td>
<td>0.52 - 0.62</td>
<td>C, M</td>
</tr>
<tr>
<td>Quartz</td>
<td>6.6 - 13</td>
<td>0.7 - 0.74</td>
<td>C, H, M, B</td>
</tr>
<tr>
<td>Vitreous silica</td>
<td>1.36</td>
<td>0.7</td>
<td>C</td>
</tr>
<tr>
<td>Olivines</td>
<td>3.16 - 5.06</td>
<td>0.55</td>
<td>C</td>
</tr>
<tr>
<td>Muscovite</td>
<td>1.7 - 2.32</td>
<td>0.76</td>
<td>C, H, M</td>
</tr>
<tr>
<td>Biotite</td>
<td>1.17 - 1.73</td>
<td>0.77</td>
<td>C, H</td>
</tr>
<tr>
<td>Clay min. (mean)</td>
<td>2.9</td>
<td></td>
<td>Q</td>
</tr>
<tr>
<td>Feldspar</td>
<td>2.3</td>
<td></td>
<td>B</td>
</tr>
<tr>
<td>Orthoclase</td>
<td>2.31 - 3.2</td>
<td>0.67 - 0.75</td>
<td>C, H, M</td>
</tr>
<tr>
<td>Plagioclase (mean)</td>
<td>2.31</td>
<td></td>
<td>H</td>
</tr>
</tbody>
</table>
Thermal conductivity of minerals

- native metals and such elements as graphite and diamond (120 W m\(^{-1}\) K\(^{-1}\))
- sulfides
- oxides (11.8)
- fluorides and chlorides (6)
- carbonates (4)
- silicates (3.8)
- sulfates (3.3)
- nitrates (2.1)
- native elements/nonmetals (selenium, sulphur ~ 0.85)

Anisotropy (see Schon, Clauser & Huenges, pdf’s on class web page)
Graphite (300 K): \(k(\text{in plane}) \sim 2000 \text{ W m}^{-1}\text{K}^{-1} \); \(k(\text{perp}) \sim 5 \text{ W m}^{-1}\text{K}^{-1}\)

Empirical correlations with density and mean atomic weight:
Horai and Simmons (1969)
\[
k = 5.5 \times 10^{-3} \rho + 1.36(20 - m_A)
\]
Thermal conductivity of composites: harder!

Heat flow along paths of least resistance!

Factor in geometry

\[ J_Q = k_{\text{eff}} \nabla T \]

(a) Parallel

(b) Series

(c) Maxwell

\[
 k_p = \sum_i \phi_i k_i \\
 k_s = \left( \sum_i \frac{\phi_i}{k_i} \right)^{-1} \\
 k = k_1 \frac{2k_1 + k_2 - 2\phi_2(k_1 - k_2)}{2k_1 + k_2 + \phi_2(k_1 - k_2)}
\]

Arithmetic mean; harmonic mean
Simple composite models
Granite conductivity & mineralogy

Quartz 7.7 W m\(^{-1}\) K\(^{-1}\)

Orthoclase 2.4 W m\(^{-1}\) K\(^{-1}\)

Plagioclase

Geometric mean

\[ k_c = \prod \phi_i \]

Fig. 12. Ternary diagram of the quartz–orthoclase–plagioclase system, with conductivity calculated from the geometric model on the basis of assumed conductivities for the three components. The shaded area denotes granite.

Uniform distribution

Fig. 13. The shaded histogram shows the distribution of conductivity of granite, calculated from Fig. 2 on the assumption of uniform distribution of granites within the zone of definition. The unshaded histogram shows the observed conductivity of 441 granites from the Canadian Shield.

Jessop

Measured, n=441
Canadian Shield
Saturation + Porosity

**Fluid saturation**

![Graph showing fluid saturation in sandstones](image)

**Bottle-necks vs. open pore space**

- White = grains (feldspar, quartz...)
- Blue/green = porosity (blue-dyed epoxy)

Clauser & Huenges, web
Simple Fracture Model

\[ k = k_m \left( 1 - D_0 \exp\left( -A \frac{p}{p_0} \right) \right) \]

(Actually need two terms to fit data, here gneiss)

Clauser & Huenges, Schön
Summary of Rock-type variations

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<tr>
<th>Rock Type</th>
<th>Paramount Factors</th>
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<td>anisotropy, dominant mineral phase</td>
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Clauser & Huenges
Thermal Conductivity of Rocks

Variability
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Different mixing models
Quartz content
Mineralogy
Fabric/structure
porosity, saturation
Conductivity mechanism
Phonons + radiative
Anisotropy

Clauser & Huenges


Google: Petrovsky and Uni. Leobden presentations.