- the response of the ice cover (i.e. its drift and deformation) to the forcing is described by the momentum balance (see also sketch in Fig. 3.1.1.1):

\[ m \frac{Du}{Dt} = \sum F = \tau_a + \tau_w + F_i + F_C + F_H \]

- where \( m \) is the mass of ice per unit area and the total time derivative of the velocity is given by

\[ \frac{Du}{Dt} = \frac{\partial u}{\partial t} + u \cdot \nabla u \]
Internal stress

- as a result of the combined action of the forces outlined above, an internal stress field may build up in an ice cover that is compact enough to deviate from the free-drift case; this force will be partially balancing (in an opposite direction for an idealized, isotropic pack, Fig. 3.1.1.2) the vector sum of the other forces, resulting in a decrease of ice velocity; the internal forces are obtained through differentiation of the stress tensor $\sigma_{ij}$:

$$F_i = \frac{\partial \sigma_{ij}}{\partial x_j}$$

Fig. 3.1.1.2.: Ice pack force balance (Hibler, 1986).
Rheological models of solids

- Linking stress \((\sigma = F/A)\) to the strain \(\varepsilon = (l-l_0)/l_0\)

- Elastic (solid): 
  \(\sigma = E \varepsilon\) 
  with Young’s modulus \(E\)

- Newtonian liquid: 
  \(\sigma = \eta \frac{d\varepsilon}{dt}\)

- Plastic (solid): 
  \(\sigma = k, \text{ for } \varepsilon > 0\) 
  \(\varepsilon = 0, \text{ for } \sigma < k\)
Rheological model of sea ice: Visco-elastic behavior

- Sea ice deformation often best described by model that takes into account elastic and viscous strain components for different stresses and strain rates:

\[ \dot{\varepsilon} = A_i \exp\left(\frac{-Q_i}{RT}\right)\sigma^n \]

for the uniaxial deformation case, where \( A_i \) is a fabric parameter (taking into account basal-plane or brine layer anisotropy), \( R \) the universal gas constant, \( T \) the absolute temperature, \( Q_i \) an activation energy and the power index \( n \) equal to roughly 3.

---

Fig. 3.2.2.2: Strain-behavior with time for different deformation regimes (Sanderson, 1988).
Accommodation of strain at the microstructural scale

- Schulson & Kuehn (1993):
  - $\varepsilon = 0\%$
  - $\varepsilon = 0.17\%$
  - $\varepsilon = 2\%$
  - $\varepsilon = 2\%$
  - $\varepsilon = 25\%$
Fig. 3.2.2.4: Stress-strain curves for frazil ice uniaxial compression tests carried out at -10°C and different strain rates (Richter-Menge, 1992).
Brittle-ductile transition in sea ice

Fig. 3.2.2.5: Brittle-ductile transition for sea-ice indentation tests (Sanderson, 1988).
Ice strength

- Yield or failure stress $\sigma_f$
Ice tensile strength

Fig. 3.2.2.6: Stress/strain-rate plot for granular sea ice (Sanderson, 1988).

Fig. 3.2.2.7: Tensile strength of granular sea ice (top) and limiting processes (bottom, Sanderson, 1988).
ICE ACTION DETERMINATION FLOW DIAGRAM

1. Structural design concept
   - Fixed or floating facility

2. Define ice action scenarios

3. Estimation of expected number of ice-structure interaction events and their duration

For each scenario:

4. Determination of action values

5. Employ extremal analysis to assign probability distribution to ice actions on the structure

6. Provide representative ice actions for design

7. Ice conditions
   - Operational conditions, ice management and disconnection capabilities

8. Ice properties and strength
   - Ice morphology
   - Environmental driving forces
ICE ACTION - CRUSHING AGAINST A VERTICAL STRUCTURE

\[ F_G = p_G \cdot h \cdot w \]

\[ p_G = C_R \left( \frac{h}{h^*} \right)^n \left( \frac{w}{h} \right)^m \]

where

- \( p_G \) is the global average ice pressure, in MPa;
- \( w \) is the projected width of the structure, in m;
- \( h \) is the thickness of the ice sheet, in m;
- \( h^* \) is a reference thickness of 1,0 m;
- \( m \) is an empirical coefficient (= -0,16);
- \( n \) is an empirical coefficient
  \[ = -0,50 + \frac{h}{5} \text{ for } h < 1,0 \text{ m} \]
  \[ = -0,30 \text{ for } h \geq 1,0 \text{ m}; \text{ and} \]
- \( C_R \) is the ice strength coefficient, in MPa.

Crushing with level ice - most common ice interaction

Vertical structure – easiest to build and transport
ICE ACTION – EXAMPLE DETERMINISTIC LOAD CALCULATION

Structure – Beaufort Sea, Vertical side GBS, 100 m wide
Ice – MYI Thickness = 4.5 m, Crushing failure
Load = ∼ 500 MN (from previous plot)

Event Occurs in Winter = No wave load, current load (10% of ice load), no wind

Total EL Action = 1.35 * Ice Action + 0.9 * Stochastically dependent actions + 0.6 * stochastically independent actions

Total EL Action = 1.35 * 500 + 0.9 *(0.1 * 500)
720 MN

Above is the left side of the equation, resistance (the right side) is determined by use of appropriate ISO Standard
Figure 3.5.11a. Crushing Failure. Section of hull structure is shown at left with stiffeners. Advancing, crushing (compressive failure) ice is shown on the right. Below the ice is water and the ice is floating on the water.

Figure 3.5.11b. Buckling Failure. Section through hull structure is shown on the right with stiffeners. Ice impacting the hull is shown on the left with a typical buckling failure. Below the ice is water and the ice floats on the water.

Masterson, 2009, in: Field techniques for sea ice research
Masterson, 2009, in: Field techniques for sea ice research
- $z$ – peak loads occurring on average $N$ times per year
- equation gives the Gumbel distribution of annual extreme maxima
- distribution is used to calculate exceedance probabilities (incl. for Monte-Carlo simulations)

Karna et al., ICETECH 2006
• Static loading forces coincides with thick ice presence
• Peak loads with both thin and thick ice as a result of crushing or buckling failure

Haas & Jochmann, POAC 2003
Microscopic ice strength

- In the simplest case, one can assume $\gamma$ and $r_a$ to be constant which would result in $V_b$ changing with $r_b$, such that the normalized failure stress would amount to

$$\frac{\sigma_f}{\sigma_0} = 1 - cV_b$$

where $c$ is a constant

- In the case of elliptical cylinders with $\epsilon = r_b/r_a$, and $F = \pi r_b^2 / \epsilon$, we obtain a root(brine volume fraction) dependence:

$$\frac{\sigma_f}{\sigma_0} = 1 - 2\sqrt{\frac{\epsilon \gamma}{\pi \beta_0}} \sqrt{V_b}$$

Fig. 3.2.2.8: Microstructural model for derivation of macroscopic sea ice strength (Weeks and Ackley, 1986).
Microscopic/macroscopic ice strength

- Richter-Menge & Jones (1993)
- Sanderson (1988)
Ice failure on microscopic scale

Fig. 3.2.2.9: Formation of wing cracks in a brittle solid under compression as a result of shear along a crack resulting in a local zone of tension (Sanderson, 1988).
Ice failure on macroscopic to floe scale

Fig. 3.2.2.10: Flexure of an ice sheet and formation of a tensional crack (note that crack does not penetrate beyond neutral fiber \((d\varepsilon = 0)\) in the center of the ice sheet (Sanderson, 1988).
ARCTIC SEA ICE MOTION DERIVED FROM SSM/I PASSIVE MICROWAVE SATELLITE (JUNE 94 TO AUGUST 95)

HAO LE
ENVIRONMENT CANADA
Rheological models of solids

- Linking stress ($\sigma = F/A$) to the strain $\varepsilon = (l-l_0)/l_0$

- Elastic (solid):
  $\sigma = E \varepsilon$
  with Young’s modulus $E$

- Newtonian liquid:
  $\sigma = \eta \frac{d\varepsilon}{dt}$

- Plastic (solid):
  $\sigma = k$, for $\varepsilon > 0$
  $\varepsilon = 0$, for $\sigma < k$
Ice failure: Regional scale

- Visco-plastic behavior with shear and bulk viscosity $\eta$, $\zeta$ and strength $P$

\[
\sigma_i = \frac{\zeta \dot{\varepsilon}_i - P}{2} \\
\sigma_{II} = \eta \dot{\varepsilon}_{II}
\]

- Hibler (1986) parameterised $P(H,a)$ as a function of the ice thickness $H$ and the areal concentration $a$:

\[
P(H,a) = P^* H \exp[-C(1-a)]
\]

<table>
<thead>
<tr>
<th>Day</th>
<th>Wind Speed (m sec$^{-1}$)</th>
<th>Wind Direction (°E of N)</th>
<th>Wind Magnitude x-direction (°E of N)</th>
<th>Air Stress Magnitude (dyn cm$^{-2}$)</th>
<th>Air Stress Magnitude (dyne cm$^{-2}$)</th>
<th>-fetch (km)</th>
<th>Strength Estimate $p^*$ (dyn cm$^{-2}$)</th>
<th>$\tau^*$ (dyne cm$^{-2}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10 Feb</td>
<td>6.0</td>
<td>255</td>
<td>1.2</td>
<td>82</td>
<td>1.2</td>
<td>0.1</td>
<td>850</td>
<td>$1.0 \times 10^8$</td>
</tr>
<tr>
<td>11 Feb</td>
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<td>255</td>
<td>0.5</td>
<td>82</td>
<td>0.5</td>
<td>0.1</td>
<td>850</td>
<td>$4.2 \times 10^7$</td>
</tr>
<tr>
<td>13 Feb</td>
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<td>235</td>
<td>0.5</td>
<td>82</td>
<td>0.4</td>
<td>0.2</td>
<td>850</td>
<td>$3.4 \times 10^7$</td>
</tr>
<tr>
<td>23 Feb</td>
<td>7.0</td>
<td>50</td>
<td>1.6</td>
<td>197</td>
<td>1.3</td>
<td>-0.9</td>
<td>600</td>
<td>$7.8 \times 10^7$</td>
</tr>
<tr>
<td>24 Feb</td>
<td>6.0</td>
<td>50</td>
<td>1.2</td>
<td>197</td>
<td>1.0</td>
<td>-0.7</td>
<td>600</td>
<td>$6.0 \times 10^7$</td>
</tr>
</tbody>
</table>

($(1 \text{ N} = 10^3 \text{ dyn}$, i.e. for ice thickness of 1 m, strength is around 10 to 100 kN m$^{-2}$)

Fig. 3.2.1.1: Map showing location of ice fields subjected to onshore wind forcing, allowing for derivation of yield stress (Pritchard, 1976).
Ice failure: Regional scale

- in a simplified approach, assuming that the ice momentum balance is controlled solely by wind and current forcing, the forces that build up within the ice cover are dependent on the wind (a) and current (w) shear stress $\tau_{w,a}$, which is given by (see Section 3.1 for more details)

$$\tau_{w,a} = c_D \rho_w |u|^2$$

- taking $c_D$ as $1.3 \times 10^{-3}$ for both the upper and lower surface and the appropriate densities for water and air, the magnitude of the shear stress is shown in Fig. 3.2.3.1

- the in-situ magnitude of the shear stress depends on the length of fetch $L$ over which the stress may build up, such that the local force per unit width of ice $F'$ is

$$F' = L \tau$$
Ice failure: Regional scale

Fig. 3.2.3.1: Surface shear stress as a function of fluid velocity (Mellor, 1986).

Based on Fig. 3.2.3.1, for an ice cover of 1 m thickness this results in values which are far below typical macroscopic ice strengths and would not result in failure even if operating over a long period of time, i.e. after accumulation of considerable strain; Parmerter and Coon (1972), however, found values for $F'$ around 1 kN m$^{-2}$ during ridge building, with $\tau$ around 0.05 N m$^{-2}$ and $L$ around 20 km.
Ice deformation at different scales

[14] The divergence, vorticity, and shear (strain rates) of each cell are computed via:

\[ \nabla \cdot u = u_x + v_y, \quad \zeta = v_x - u_y, \]
\[ e = \left[ (u_x - v_y)^2 + (u_y + v_x)^2 \right]^{1/2} \quad (1) \]

\( u_x, u_y, v_x, v_y \) are the spatial gradients in ice motion computed using a line integral around the boundary of each cell (\( \sim 10 \) km on a side). The line segments connecting the four vertices of a cell define the boundaries. \( \nabla \cdot u \) is a measure of the rate of area change, \( \zeta \) is the principal measure of rotation rate, and \( e \) is the scalar magnitude of shear.

Kwok, JGR, 2006
Ice deformation at different scales

The divergence, vorticity, and shear (strain rates) of each cell are computed via:

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\nabla \cdot \mathbf{u} = u_x + v_y, \quad \zeta = v_x - u_y,
$$

$$
ee = \left[ (u_x - v_y)^2 + (u_y + v_x)^2 \right]^{\frac{1}{2}}
$$

(1)

$u_x, u_y, v_x, v_y$ are the spatial gradients in ice motion computed using a line integral around the boundary of each cell ($\sim 10$ km on a side). The line segments connecting the four vertices of a cell define the boundaries. $\nabla \cdot \mathbf{u}$ is a measure of the rate of area change, $\zeta$ is the principal measure of rotation rate, and $e$ is the scalar magnitude of shear.

Figure 7. Mean monthly deformation in the perennial and seasonal ice zones. (a) Fractional coverage of deformed cells. (b) Mean monthly shear. (c) Change in deformed ice coverage between November and April. Quantities in the seasonal and perennial ice zones are plotted as dashed and solid lines, respectively.
Ice deformation at different scales


Fig. 27. Photographs showing comb cracks on scales large and small: (a) satellite image of the sea ice cover off the coast of Greenland, 25 April 1976 (from Ref. [169]). (b) Suggested shear loading of the macrocolumns created by the secondary cracks. (c) Cracks in freshwater granular ice of ~10 mm grain size loaded triaxially across-column ($R \sim 0.05$) at $-40^\circ$C at $10^{-3}$ s$^{-1}$.
Continuum/granular behavior of sea ice

- Schulson (2001): granular material and continuum aspects of ice deformation behavior

Fig. 26. Schematic sketch of two types of deformation: type-I depicts ice as a continuum; type-II, as a granular material. A fault/lead develops during terminal failure (b,c), creating gouge (c) across which frictional sliding occurs at lower applied stresses. Eventually, the fault/lead heals or the wind changes direction (d) and the ice again behaves as a continuum.