Sea ice dynamics

- Wind forcing
  - Coriolis force
  - Inertial force
  - Form drag
  - Frictional drag

- Ocean currents
- Sea-surface slope

- Large-scale rheology
- Form/frictional drag
- Internal stress/forces
- Frictional drag

- Long-wave flux
  - Turbulent flux
  - Conductive heat flux

- Shortwave flux
  - Solar heating of ice and ocean (primary production by phytoplankton)
  - Ocean-ice heat flux

- Ocean currents
Deformation processes

rafting

ridging

may be >10 m

thin ice

thick ice

ice floes

hummocks

weathering

fractures

Adapted from Environment Canada
Fractures
Rafting without fracture: up to 17 cm ice thickness (Parmerter, 1975)
Pressure ridge

Photo (near-IR): Chris Petrich
Haas (2009)

Generation of open water

-> new ice growth

Pressure ridge formation
Rafting of relatively thick ice (0.5 to 0.9 m) is possible if rubble accumulates at the floe edge.
Fig. 16. Sketch showing the features of an "average" first-year ridge.

Fig. 17. Sketch showing the features of an "average" multi-year ridge.

Timco & Burden (1997)
Fig. 4.3  Aerial photographs of typical sea ice types and conditions, and graphs of their corresponding ice thickness distributions (given as probability density functions – PDFs – of total (ice plus snow) thickness): (a) first-year ice in the Weddell Sea, (b) heavily deformed multiyear ice in the Lincoln Sea and (c) second-year ice at the North Pole in summer.
Change in ice thickness distribution in 40 days

Figure 5. Comparison of modeled (thick solid line) and observed (thin solid line) changes in ice volume coverage: (a) model results with ridging only and (b) model results with both rafting and ridging.

Babko et al. (2002)
Ice thickness distribution

- thus the mean thickness over a region R of area A is given by

\[ \hat{H}(x, y) = \iint_R h \frac{da(h)}{A} = \iint h g(h) \, dh \]

where \( g(h) \, dh \) is the ice thickness distribution defined as the fraction of R with thickness between \( h \) and \( h+dh \) (and has units \( L^{-1} \))

- thus, if \( h(x) \) is assumed to be a stochastic process with \( h \) at a given position \( x \) as a random variable, the thickness distribution is described the probability density function

\[ P = \int_a^b g(h) \, dh \]

that yields the probability \( P \) of encountering \( a<h<b \)

- mean and variance for \( H \) as measured along a profile \( L \) are given by

\[ \hat{H} = \frac{1}{L} \int_0^L h(x) \, dx \]

\[ \text{var}(\hat{H}) = E\left( \hat{H} - H \right)^2 = \frac{1}{L^2} \iint_L E[h(x_1) - H]E[h(x_2) - H] \, dx_1 \, dx_2 \]
Ice thickness distribution

Fig. 3.3.1.1: Ice draft distributions in the Fram Strait sector of the Arctic Ocean (Wadhams, 1995).
Ice thickness distribution

- thus the mean thickness over a region \( R \) of area \( A \) is given by

\[
\hat{H}(x, y) = \iint_R h \frac{da(h)}{A} = \iint_R h \frac{g(h)}{\Sigma(h)} \, dh
\]

where \( g(h) \, dh \) is the ice thickness distribution defined as the fraction of \( R \) with thickness between \( h \) and \( h+dh \) (and has units \( L^{-1} \))

- thus, if \( h(x) \) is assumed to be a stochastic process with \( h \) at a given position \( x \) as a random variable, the thickness distribution is described the probability density function

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\]

\[
\text{var}(\hat{H}) = E(\hat{H} - H)^2 = \frac{1}{L^2} \int_L \int \left[ h(x_1) - H \right] \left[ h(x_2) - H \right] \, dx_1 \, dx_2
\]

with \( s = |x_1 - x_2| \) and the autocovariance function \( C_h(s) \), one obtains

\[
\text{var}(\hat{H}) = \frac{2}{L^2} \int_0^L \left( 1 - \frac{s}{L} \right) C_h(s) \, ds
\]
Table 3.3.1.1: Standard deviation of ice draft estimate for different profiles lengths $L$

<table>
<thead>
<tr>
<th>$L$, km</th>
<th>$(\text{var } \hat{H})^{1/2}$, m</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0.38</td>
</tr>
<tr>
<td>50</td>
<td>0.17</td>
</tr>
<tr>
<td>100</td>
<td>0.12</td>
</tr>
<tr>
<td>500</td>
<td>0.05</td>
</tr>
<tr>
<td>1000</td>
<td>0.04</td>
</tr>
</tbody>
</table>

$$\text{var}(\hat{H}) = \frac{\sigma^2}{n} = \frac{C_h(0)}{n}$$
Ice thickness distribution theory

Fig. 3.3.1.3: Schematic view of different terms of the ice thickness distribution function and their contribution to the shape of the ice thickness pdf.

- the evolution of the ice thickness distribution $g(h,X,t)$ with time has been described by Thorndike et al. (1975) as (see also Fig. 3.3.1.3)

$$\frac{\partial g}{\partial t} = \psi - div(Ug) - \frac{\partial (fg)}{\partial h} + \phi$$
Ice thickness distribution theory

- here the flux divergence $\text{div}(Ug)$ accounts for convergence/divergence $-g \text{div}(U)$ and advection $-U \cdot \text{grad}(g)$; the thermal growth $f = dh/dt$ carries ice of a given thickness to another thickness value, yielding a term analogous to the flux divergence

$$\frac{\partial g}{\partial t} = - \frac{\partial (fg)}{\partial h}$$

with

$$\frac{\partial G}{\partial t} = - f \frac{\partial G}{\partial h}$$

in its integral form; while the ice-growth function is well constrained through thermodynamic modelling, the melting is less well understood and at present dealt with empirically (see Fig. 3.3.1.3)

- lateral melting $\phi$ reduces the area of ice with a concurrent increase in the amount of open water such that the amount of heat $Q$ available for melting ice of density $\rho$ and thickness $H$ (with $L$ latent heat of melting and $\delta(h)$ Dirac's delta function) is

$$\phi = \frac{Q}{\rho LH} \left[ \delta(h) - g(h) \right]$$

with

$$H = \int_0^\infty hg \, dh$$

- the redistribution function $\psi$ describes ridging and opening of leads; while the former process leaves the mean thickness of ice unchanged, while the changes in fractional ice/water area due to the latter are compensated by ice import through convergence; in defining $\psi$, one resorts to analyses of the ridging process and the large-scale ice rheology as outlined in Section 3.3.2, with $\psi$ depending on the strain rate, the areal losses and gains of thin and thick ice $\mu_{L,G}$ and an empirical coefficient $\alpha$ dependent on the stress regime
# Ice thickness distribution theory

Table 3.3.1.2: Annual average of thickness distribution function terms (integrated over thickness range specified at top; units are $\% d^{-1}$ and $\%$ for the thickness distribution)

<table>
<thead>
<tr>
<th>Terms in (8.3b) and (8.5)</th>
<th>0-0.1 m</th>
<th>0.1-1.6 m</th>
<th>1.6-6.4 m</th>
<th>6.4-25.6 m</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thermal flux gradient $- \frac{\partial}{\partial h}(fg)$</td>
<td>-0.59</td>
<td>0.43</td>
<td>0.013</td>
<td>-0.003</td>
</tr>
<tr>
<td>Open water production $</td>
<td>\varepsilon</td>
<td>\alpha_o \delta$</td>
<td>0.73</td>
<td></td>
</tr>
<tr>
<td>Ridging loss $-</td>
<td>\varepsilon</td>
<td>\alpha_r u_L$</td>
<td>-0.14</td>
<td>-0.44</td>
</tr>
<tr>
<td>Ridging gain $</td>
<td>\varepsilon</td>
<td>\alpha_r u_G$</td>
<td>0.00</td>
<td>0.01</td>
</tr>
<tr>
<td>Import $-g \text{ div } u$</td>
<td>0.00</td>
<td>0.00</td>
<td>-0.028</td>
<td>-0.009</td>
</tr>
<tr>
<td>Rate of change of $g$</td>
<td>0.00</td>
<td>0.00</td>
<td>0.000</td>
<td>0.002</td>
</tr>
<tr>
<td>Thickness distribution $b \int_a^g \text{d}h$</td>
<td>2.9</td>
<td>20.0</td>
<td>58.9</td>
<td>18.1</td>
</tr>
</tbody>
</table>
Figure 5: Maps of the Arctic and Southern Ocean, showing model results of mean ice drift speed and direction (vectors) as well as mean ice thicknesses (colours), calculated for the winters of 1985 to 1993. (a: Koeberle et al 2003  b: Timmermann et al., 2002). Note that thickness intervals are irregular.
Measurements of sea ice thickness

Satellite altimeter (RADAR or LASER)

Electromagnetic induction & LASER altimeter (EM Bird)

Drilling

• Autonomous Underwater Vehicle (AUV)…
• Moorings…
  …with upward-looking sonar

Submarine with upward-looking sonar

*all but moorings used at APLIS, Beaufort Sea, April 2007*
## Ice thickness measurement techniques

<table>
<thead>
<tr>
<th>Method</th>
<th>Resolution</th>
<th>Spatial</th>
<th>Thickness</th>
<th>Coverage</th>
<th>Reliability</th>
<th>Cost</th>
<th>Logist. effort</th>
</tr>
</thead>
<tbody>
<tr>
<td>Drilling</td>
<td></td>
<td>4</td>
<td>4</td>
<td>1</td>
<td>4</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>Impulse radar</td>
<td></td>
<td>4</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>3</td>
<td>2</td>
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<tr>
<td>(Airborne) EM</td>
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<td>2</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>2</td>
<td>3</td>
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<tr>
<td>Submarine sonar</td>
<td></td>
<td>4</td>
<td>4</td>
<td>3</td>
<td>4</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>UL sonar</td>
<td></td>
<td>2</td>
<td>3</td>
<td>1</td>
<td>3</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>Laser altimetry (airb.)</td>
<td></td>
<td>4</td>
<td>2*</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
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<tr>
<td>Satellite rem. Sens. +</td>
<td></td>
<td>3</td>
<td>2</td>
<td>4</td>
<td>2</td>
<td>2</td>
<td>4</td>
</tr>
</tbody>
</table>

* only true for Arctic
+ combination of laser/radar altimetry, SAR, IR, optical and passive microwave techniques