1. From the weak growth (damping) rate approximation, we have learned that the sign of $\gamma$ is given by the sign of $\frac{\partial F_0(v_z = \omega/k)}{\partial v_z}$ (for positive $k$) and thus the wave is damped (stable) if the slope is negative, and it is amplified (unstable) if the slope is positive. This suggests that instability can happen only when $\frac{\partial F_0(v_z = \omega/k)}{\partial v_z} > 0$, and thus the distribution function $F_0(v_z)$ has more than a single hump.

2. This turns out to be true even without the weak growth rate approximation. This is called the Gardner’s theorem. In other words, Gardner’s theorem shows that a single-humped velocity distribution is stable.

3. However, a distribution having more than one hump is not necessarily unstable.

4. One way to see if instability can occur is to map the marginal stability line ($\gamma = 0$) on the complex $p$-plane onto the complex $D$-plane, where $D$ is the dispersion function. If this closed contour encloses the $D = 0$ point, then it is unstable. Otherwise, it is stable. This is called the Nyquist criterion.

5. Applying the Nyquist criterion, one can derive an even simpler condition for instability: 
$$\int_{-\infty}^{\infty} \frac{F_0(v_z) - F_0(V_j)}{(v_z - V_j)^2} dv_z > 0,$$ 
where $F_0(v_z)$ is a minimum at $v_z = V_j$. This is called the Penrose condition. On the other hand, the wave is stable if 
$$\int_{-\infty}^{\infty} \frac{F_0(v_z) - F_0(V_j)}{(v_z - V_j)^2} dv_z < 0$$ 
for every $V_j$. 