1. The linearized Vlasov-Poisson equations can now be solved by using Fourier transform in space and Laplace transform in time, given an initial value of $f(0)$.

2. The Laplace transform of the electric potential $\tilde{\Phi}(k,p)$ now satisfies an inhomogeneous equation (with the right hand side depending on the initial condition), unlike the homogeneous equation using Fourier transform in the Vlasov approach.

3. The solution $\tilde{\Phi}(k,p)$ is in the form of $\tilde{\Phi}(k,p) = N(k,p)/D(k,p)$, with $N(k,p)$ depending on the initial condition.

4. The locations of the poles in $\tilde{\Phi}(k,p)$ are the solutions of $D(k,p) = 0$, which serves in a role likes the dispersion relation (but it is not strictly speaking the dispersion relation).

5. Both $N(k,p)$ and $D(k,p)$ depend on integrals in $v_z$ with a factor $(v_z - ip/k)^{-1}$ in the integrands, similar to the $D(k,\omega)$ in the dispersion relation based on the Vlasov approach. The main difference is now $p$ is complex and so the integrals are well defined unless $\Re p = 0$.

6. However, in order to maintain a continuous $\tilde{\Phi}(k,p)$ throughout the complex $p$-plane, the integration contour in $v_z$ has to be deformed as $p$ moves from $\Re p > 0$ to $\Re p < 0$.

Homework #10 (due Monday, November 30th, before class): Problem # 8.2, 8.3, 8.4, 8.5, and a 1994 comprehensive exam question in the next page.
Problem Consider a plasma which consists of electrons and positrons. Both electrons and positrons are assumed cold \( T_e = T_p = 0; \ m_e = m_p \). Positrons stream with a velocity \( v_p = u_p \hat{x} \), while electrons stream with a velocity \( v_e = -u_e \hat{x} \). Consider the electrostatic waves propagating in the \( x \) direction. The density for each species is \( n_o \).

(40) a) Use the following equation obtained from the kinetic theory

\[
\mathcal{E} (k, \omega) = 1 - \sum_{\alpha} \frac{\omega_{p\alpha}^2}{k^2} \int_{\alpha} du \frac{\partial}{\partial u} F_{\alpha} (u) \frac{(u)}{(u - \omega/k)} = 0
\]

to obtain the dispersion relation for the electrostatic waves in the electron-positron plasma system.

(40) b) Use the fluid equations to obtain the dispersion and compare the result with that in (a).

(20) c) Obtain the condition for the instability to occur and the growth rate for the dispersion relation of either part (a) or part (b).

Fluid Equations:

\[
\begin{aligned}
\frac{\partial}{\partial t} n_\alpha + \nabla \cdot (n_\alpha v_\alpha) &= 0 \\
m_\alpha n_\alpha \left[ \frac{\partial}{\partial t} v_\alpha + (v_\alpha \cdot \nabla) v_\alpha \right] &= -\nabla P_\alpha + q_\alpha n_\alpha E \\
\nabla \cdot E &= 4\pi \sum q_\alpha n_\alpha
\end{aligned}
\]