1. The third order moment equations are obtained by integrating the Vlasov equation multiplied by $vv$ over velocity space. This produces nine equations with six of them being independent. The simplest form of equation is to sum over all the diagonal terms (trace), which is the same as integrating the Vlasov equation multiplied by $v^2$ over velocity space.

2. The resulting equation is the energy equation (conservation of energy).

3. The time evolution of the kinetic energy density $W_s$, which is a second moment, depends on the kinetic energy flux $Q_s$, which is a third moment.

4. In general, the evolution of the $n^{th}$ moment depends on the $(n+1)^{th}$ moment, and thus this forms an infinite hierarchy of equations. This is called the closure problem. To form a closed set of $n$ equations, assumed relations between the $(n+1)^{th}$ order moments to other lower order moments have to be applied.

5. To truncate up to just the first moment equation (the equation of motion), the pressure tensor $\mathbf{P}_s$ has to be related to the average velocity and/or number density. This is called the equation of state.

6. One form of equation of state can be expressed as $\frac{d(p_s/n_s^2)}{dt} = 0$ if the pressure tensor is isotropic with $\mathbf{P}_s = p_s \mathbf{I}$. $\gamma = 5/3$ is usually chosen for an adiabatic case in 3D. $\gamma = 1$ is for an isothermal case, and $\gamma \to \infty$ is for an incompressible case.

7. For the anisotropic $\mathbf{P}_s$, the perpendicular ($p_{s\perp}$) and parallel ($p_{s\parallel}$) pressures are different. A commonly used equation of state is the Chew-Goldberger-Low (CGL) equation of state $\frac{d(p_{s\perp}/n_s B)}{dt} = 0$ and $\frac{d(p_{s\parallel}B^2/n_s^3)}{dt} = 0$. They are related to the first and second adiabatic invariants.

Homework #5 (due Friday, October 16th, before class): Problem # 5.2, 5.3, 5.4, 5.5