1. When the distribution function is evolving through the Vlasov equation, a phase space volume carried by phase space trajectories remains the same, i.e., “incompressible” in the phase space.

2. Since the number of particles within a phase space volume is also unchanged, $f_s$ is a constant along a trajectory in the phase space, i.e.,

$$\frac{df_s}{df} + v \cdot \nabla f_s + \frac{q_s}{m_s} (E + v \times B) \cdot \nabla_s f_s = 0$$

3. If $C_1(r(t), v(t)), C_2(r(t), v(t)), \ldots$ are constants of motion, i.e.,

$$\frac{dC_1[r(t), v(t)]}{dt} = 0, \ldots \text{etc, any function of the form } f[C_1(r, v), C_2(r, v), \ldots] \text{ is a solution of the Vlasov equation.}$$

4. Energy is an example of constant of motion.

5. We can take moments (multiply by powers of $v$ and integrate over the $v$-space) of the Boltzmann equation and obtain moment equations.

6. The zeroth order moment equation is simply the continuity equation for each species

$$\frac{\partial n_s}{\partial t} + \nabla \cdot (n_s U_s) = 0.$$ 

7. The first order moment equation is the momentum equation (equation of motion, Newton’s second law):

$$m_s n_s \frac{dU_s}{dt} = m_s n_s \left[ \frac{\partial U_s}{\partial t} + U_s \cdot \nabla U_s \right] = n_s e_s [E + U_s \times B] - \nabla \cdot \vec{P}_s + \frac{\delta \rho_s}{\delta t}$$

8. The first time derivative in the above equation is called the convective derivative.