Non-Equilibrium and Current Sheet Formation in Line-Tied Magnetic Fields: Heating of the Solar Corona

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Outline

• Introduction
  ◆ Solar corona: heating problem
  ◆ Parker's Model (1972)
• A theorem on Parker's model [Ng & Bhattacharjee, 1998]
• Possible topology of the current sheet
• Evidence from numerical simulations
• Reconnection without magnetic nulls
• Future directions
• Conclusion
The Coronal Heating Process

The Sun's outer atmosphere (the Corona) is hotter than 1,000,000°C (1,800,000°F) while the visible surface has a temperature of only about 6000°C (10,000°F). The nature of the processes that heat the corona, maintain it at these high temperatures, and accelerate the solar wind is a third great solar mystery.

Usually temperatures fall as you move away from a heat source. This is true in the Sun's interior right up to the visible surface. Then, over a relatively small distance, the temperature suddenly rises to extremely high values. Several mechanisms have been suggested as the source of this heating but there is no consensus on which one, or combination, is actually responsible.

http://science.nasa.gov/ssl/PAD/solar/quests.htm
Solar corona: basic parameters

<table>
<thead>
<tr>
<th></th>
<th>photosphere</th>
<th>corona</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temperature</td>
<td>$\sim 5 \times 10^3$ K</td>
<td>$\sim 10^6$ K</td>
</tr>
<tr>
<td>Density</td>
<td>$\sim 10^{23}$ m$^{-3}$</td>
<td>$\sim 10^{12}$ m$^{-3}$</td>
</tr>
<tr>
<td>Time scale</td>
<td>$\sim 10^4$ s</td>
<td>$\sim 20$ s</td>
</tr>
</tbody>
</table>

Magnetic fields ($10$–$100$ G) --- role in heating?

Two main theories:

- Alfvén wave
- current sheets
Solar coronal magnetic fields


2003 James Clerk Maxwell Prize to Eugene N. Parker
University of Chicago

Citation:
"For seminal contributions in plasma astrophysics, including predicting the solar wind, explaining the solar dynamo, formulating the theory of magnetic reconnection, and the instability which predicts the escape of the magnetic fields from the galaxy."
[PR1.001] Tangential Discontinuities in Untidy Magnetic Topologies

Eugene N. Parker (Department of Physics, University of Chicago, Chicago, Illinois)

The tenuous corona of the Sun, and most other stars, emits X-rays from regions of million degree gas confined in re-entrant (bipolar) magnetic fields. The basic question is the heat input responsible for the million degree temperatures. There seems to be a direct association of heat input with magnetic field. We suggest that an important contribution comes from the dissipation of magnetic free energy of the generally untidy interwoven internal topology of the bipolar magnetic fields. The dissipation is provided by the trend toward surfaces of tangential discontinuity (STD’s) in the magnetic field as the field relaxes toward static equilibrium. The incipient STD’s become sites of rapid reconnection. The tendency to STD’s can be shown from the force-free equilibrium equations (for a magnetic field embedded in a gas with uniform pressure and infinite electrical conductivity). The equations have two families of complex characteristics and one family of real characteristics, viz. the field lines, giving them this curious property. The STD’s are an essential part of the equilibrium solutions because the torsion in the field is constrained to be constant along the real characteristics while the field lines wrap first one way and then the other at different locations along the field. This contradiction is resolved by the formation of STD’s. The optical analogy for the extension of field lines through an inhomogeneous field illustrates the formation of gaps in the flux surfaces, leading to the STD’s.
Stellar corona

X-ray image taken by Chandra (http://chandra.harvard.edu/photo/cycle1/0054/0054_xray_72l.jpg) showing about a thousand X-ray emitting stars in the Orion Nebula Star Cluster.
Galaxy

Chandra's X-ray image (blue) combined with Hubble's optical image (red and green) to of the spiral galaxy NGC 3079 showing filaments consisting of warm and hot (about ten million degrees Celsius) gas near the center. (From http://chandra.harvard.edu/photo/2003/ngc3079/ngc3079_comp.jpg).
Galaxy cluster

Chandra color composite image of the galaxy cluster RDCS 1252.9-2927 shows the X-ray (purple) light from 70-million-degree Celsius gas in the cluster, and the optical (red, yellow and green) light from the galaxies in the cluster. (From http://chandra.harvard.edu/photo/2004/rdcs1252/rdcs1252_xray_opt.jpg.)
Heating by current sheets

- Magnetic diffusion equation: \( \frac{\partial B}{\partial t} = \eta \nabla^2 B \)

- Magnetic diffusion time \( \tau_d = \frac{L^2}{\eta} \)

- For resistivity \( \eta \sim 10^{-6} \text{ km}^2\text{s}^{-1} \), \( L \sim 10^7 \text{ m} \) (\( \sim 1\% R_s \)), \( \tau_d \sim 10^6 \text{ years}! \)

- Requires \( \tau_d \sim 10^4 \text{ s} \), then \( L \sim 10^2 \text{ m} \), i.e., current sheet (note \( J = \nabla \times B \) large)
Tangential discontinuities as current sheets

\[ \mathbf{J} = \nabla \times \mathbf{B} \]

- Total current \( I = \int \nabla \times \mathbf{B} \cdot d\mathbf{a} = \oint \mathbf{B} \cdot d\mathbf{l} \neq 0 \)
  even for \( L \rightarrow 0 \).
Tangential discontinuities in a magnetic non-equilibrium

- Photospheric motion has time scale $\tau \sim 10^4$ s much longer than Alfvén time.
- Corona in quasi-equilibrium --- most of the time.

Parker’s model (1972): Non-equilibrium driven by complex photospheric motions.

i.e. surfaces of tangential discontinuities separating regions of quasi-equilibrium
Parker's Model (1972)

Straighten a curved magnetic loop

Photosphere

Footpoint Twisting

Smooth Uniform Field

Non equilibrium with Current Sheet (Tangential Discontinuity)
Objections to Parker's model

- van Ballegooijen (1985)
- Longcope & Strauss (1994)
- Cowley, Longcope & Sudan (1997)
- and others

- An equilibrium can always be found for smooth footpoint motions.
- Tangential discontinuities cannot exist for a simple sheet geometry based on smooth footpoint driving.
van Ballegooijen (1985)

Current sheet => singular footpoint mapping
Reduced MHD equations

\[
\begin{align*}
\frac{\partial \Omega}{\partial t} + [\phi, \Omega] &= \frac{\partial J}{\partial z} + [A, J] + \nu \nabla^2 \Omega \\
\frac{\partial A}{\partial t} + [\phi, A] &= \frac{\partial \phi}{\partial z} + \eta \nabla^2 A
\end{align*}
\]

\[B = \hat{z} + B_\perp = \hat{z} + \nabla_\perp A \times \hat{z} \quad \text{--- magnetic field,}\]

\[v = \nabla_\perp \phi \times \hat{z} \quad \text{--- fluid velocity,}\]

\[\Omega = -\nabla_\perp^2 \phi \quad \text{--- vorticity,}\]

\[J = -\nabla_\perp^2 A \quad \text{--- current density ,}\]

\[\eta \quad \text{--- resistivity, } v \quad \text{--- viscosity,}\]

\[[\phi, A] \equiv \phi_y A_x - \phi_x A_y\]
Magnetostatic equilibrium

\[ \frac{\partial J}{\partial z} + [A, J] = 0, \text{or} \quad B \cdot \nabla J = 0 \]  

(current density fixed on a field line)

with \( \phi = \eta = 0 \). Field-lines are tied at \( z = 0, L \).

c. f. 2D Euler equation

\[ \frac{\partial \Omega}{\partial t} + [\phi, \Omega] = 0 \]

\[ A \leftrightarrow \phi, \quad J \leftrightarrow \Omega, \quad z \leftrightarrow t \]

Existence theorem: If \( \Omega \) is smooth initially, it is so for all Time. However, Parker problem is not an initial value problem, but a two-point boundary value problem.
Footpoint Mapping

\[ \mathbf{x}_\perp(z) = \mathbf{X}[\mathbf{x}_\perp(0), z], \quad \mathbf{x}_\perp(L) = \mathbf{X}[\mathbf{x}_\perp(0), L], \]

with \( \frac{dX}{dz} = \frac{\partial A}{\partial y}(X, Y, z), \quad \frac{dY}{dz} = -\frac{\partial A}{\partial x}(X, Y, z) \)

Identity mapping: \( \mathbf{x}_\perp(L) = \mathbf{x}_\perp(0) \)

e. g. uniform field \( \mathbf{B} = \hat{z} \) or \( A = \text{const} \)

Parker’s argument: non-equilibrium developed if footpoint mapping becomes too complicated.
van Ballegooijen (1985)

If $\phi(L)$ is smooth, smooth equilibrium $A(x)$ can always be found by solving:

$$
\frac{\partial}{\partial z} \left\{ \frac{\partial \Omega}{\partial z} + \nabla^2_{\perp} [\phi, A] \right\} + \left[ \frac{\partial \phi}{\partial z} + [A, \phi], J \right] 
+ \left[ A, \frac{\partial \Omega}{\partial z} + \nabla^2_{\perp} [\phi, A] \right] = 0
$$

and

$$
\frac{\partial A}{\partial t} + [\phi, A] = \frac{\partial \phi}{\partial z}
$$
A theorem on Parker's model

For any given footpoint mapping connected with the identity mapping, there is at most one smooth equilibrium.

A proof for RMHD or Strauss equations, periodic boundary condition in $x_\perp$.

(Ng & Bhattacharjee, Phys. Plasmas 1998)
Implication

An unstable but smooth equilibrium cannot relax to a second smooth equilibrium, hence must have current sheets.
Recent generalization

• J. J. Aly [2004] has generalized part of our proof to the case of force-free equilibria, described by the full MHD equations.

• He has shown that the only smooth force-free magnetic field topologically equivalent to the uniform axial field $B_0$ in a cylindrical domain is the field $B_0$ itself.
Simulations of Parker's model

♦ Start with a uniform $\mathbf{B}$ field
♦ Apply constant footpoint twisting
  $\phi(x_\perp, 0) = 0, \phi(x_\perp, L) = \phi_0(x_\perp)$ with
  $[\phi_0, \nabla^2_\perp \phi_0] \neq 0$
♦ Current layers appear after large distortion.
♦ Quasi-equilibrium at first, becomes unstable
♦ $J$ grows faster in the middle $\Rightarrow$ non-equilibrium
Simulations of Parker's model

$t = 118.5$

**bottom**

$J_{\text{max}} = 9.85508$

$J_{\text{min}} = -9.48398$

$z = 1.00000$

**middle**

$J_{\text{max}} = 9.85290$

$J_{\text{min}} = -9.46450$

$z = 8.00000$

$t = 150.5$

**bottom**

$J_{\text{max}} = 16.4100$

$J_{\text{min}} = -12.2663$

$z = 1.00000$

**middle**

$J_{\text{max}} = 35.2654$

$J_{\text{min}} = -16.2217$

$z = 8.00000$
Simulations of Parker's model

<table>
<thead>
<tr>
<th>$t$</th>
<th>$J_{\max}$</th>
<th>$J_{\max\ 0}$</th>
<th>$\langle q^2 \rangle$</th>
</tr>
</thead>
<tbody>
<tr>
<td>118.5</td>
<td>9.91</td>
<td>9.86</td>
<td>0.233</td>
</tr>
<tr>
<td>131.6</td>
<td>16.5</td>
<td>10.9</td>
<td>0.930</td>
</tr>
<tr>
<td>145.6</td>
<td>28.3</td>
<td>14.6</td>
<td>4.361</td>
</tr>
<tr>
<td>150.5</td>
<td>35.3</td>
<td>16.4</td>
<td>21.98</td>
</tr>
</tbody>
</table>

with $q \equiv \partial J / \partial z + [A, J]$. 
Objections to Parker's model

- van Ballegooijen (1985)
- Longcope & Strauss (1994)
- Cowley, Longcope & Sudan (1997)
- and others

- An equilibrium can always be found for smooth footpoint motions.
- Tangential discontinuities cannot exist for a simple sheet geometry based on smooth footpoint driving.
Force-free equilibrium \((\int \mathbf{B} \cdot d\mathbf{l} = 0)\)

\[
\int_{1} B_1(l) dl - \int_{2} B_1(l) \cdot \hat{B}_2(l) dl = 0
\]

\[
\int_{2} B_2(l) dl - \int_{1} B_2(l) \cdot \hat{B}_1(l) dl = 0
\]

\[
\int_{1+2} B(l)[1 - \hat{B}_2(l) \cdot \hat{B}_1(l)] dl = 0
\]

therefore, no discontinuity.
More general topology

Parker's optical analogy (1990)
Possible topology

\[ \int_{1} B_1(l) dl - \int_{2} B_1(l) \cdot \hat{B}_2(l) dl = 0 \]
\[ \int_{2} B_2(l) dl - \int_{1} B_2(l) \cdot \hat{B}_1(l) dl \neq 0 \]
\[ \int_{1+2} B(l) [1 - \hat{B}_2(l) \cdot \hat{B}_1(l)] dl \neq 0 \]

May have tangential discontinuities
Possible topology

\[ z = 0 \]
Main current sheet
Current layers in simulations

An example using 3D coalescence instability

- Initial equilibrium:
  \[ A_0 = \bar{A} \sin(2\pi x) \sin(2\pi y) \]

- \( \phi = 0 \) (line-tied) at \( z = 0 \) and \( z = L \)

- Unstable for \( 4\pi^2 \bar{A} L > 10.81 \)

- Energy dissipated by viscosity

- \( J_{\text{max}} \) increases with \( t \) and resolution

- \( J_{\text{max}} \) larger in middle

  --- non-equilibrium increases with \( t \)
3D coalescence instability

A

J

Top Middle Bottom
3D coalescence instability

$J$ contours

$z = 0$
3D coalescence instability

$J$ contours

$z = 0$
3D coalescence instability

$z = 0$

$J$ contours
c. f. recent results in QSLs

• Qusai-separatrix layers (QSLs) [Titov et al. 2004]:

\[ Q = \frac{(\partial X_- / \partial x_+)^2 + (\partial X_- / \partial y_+)^2 + (\partial Y_- / \partial x_+)^2 + (\partial Y_- / \partial y_+)^2}{|(\partial X_- / \partial x_+)(\partial Y_- / \partial y_+) - (\partial X_- / \partial y_+)(\partial Y_- / \partial x_+)|} \]

• Can determine $Q$, independent of using the forward or backward footpoint mapping.

• Strong current layers can form in high $Q$ region, where magnetic reconnection can occur without magnetic nulls.
c. f. recent results in QSLs

• A typical constant $Q$ surface [Titov et al. 2004] where strong current layers tend to form.

• Note that the QSL rotates 90 degrees from the bottom to the top.

• Qualitatively consistent with the geometry of our predicted current sheet.

From [Titov et al. 2004]
c. f. recent results in QSLs
Reconnection without nulls

Formation of a true singularity (current sheet) thwarted by the presence of dissipation and reconnection.

Classical reconnection geometries involve:
(i) closed field lines in toroidal devices
(ii) magnetic nulls in 3D

Parker’s model is interesting because it fall under neither class (i) or (ii).
Reconnection without nulls

Magnetic field lines before reconnection

Magnetic field lines after reconnection

$z = 0$
Reconnection without nulls

Magnetic field lines before reconnection

Magnetic field lines after reconnection

\[ z = 0 \]

Top view
Reconnection without nulls

Magnetic field lines before reconnection

Magnetic field lines after reconnection
Future directions

- High-resolution simulations using more advanced numerical methods --- spectral elements, Adaptive Mesh Refinement (AMR).
- Extending theoretical treatments --- force free equilibrium, full MHD/Hall MHD, realistic geometry.
- Couple with research on magnetic reconnection.
- Related to observations of the corona, or interplanetary magnetic clouds.
- Development of MHD turbulence/further enhancement in heating/particles acceleration.
Future directions

- Related to observations of the corona, or interplanetary magnetic clouds.

A sketch (not to scale) of the magnetic cloud prior to its arrival at Earth. (from [Farrugia et al. 1993]).
Figure 6: A PMS associated with an ejecta. The figure shows the interface region between the October 18-19, 1995 magnetic cloud and the trailing faster corotating stream. From top to bottom: density, bulk speed, temperature, dynamic pressure, $\epsilon$ parameter, total field and its components in principal axes coordinates. This structure resulted in storm and substorm activity.
More discontinuities are re-classified as tangential discontinuities

[Horbury et al., 2000]

Figure 1. Scatterplot of relative field magnitude change across discontinuities against the fraction of field threading the normal plane. Data are shown based on normal estimates from three spacecraft timings (solid circles) and Wind magnetic field minimum variance vectors (open squares).
Future directions

- Development of MHD turbulence/further enhancement in heating/particles acceleration.

  Reconnection generates turbulence
  
  Turbulence enhances reconnection rate

Tie up with my another main research area of MHD turbulence.
Current sheet formation

- 2D MHD high-resolution turbulence simulations [Ng et al., 2003]
- Intense current “sheets”
- Local anisotropy along local $\mathbf{B}$
Current sheet formation

- 2D MHD high-resolution turbulence simulations [Ng et al., 2003]
- Intense current “sheets”
- Local anisotropy along local $\mathbf{B}$

![Diagram of current density $J$](image)
Current sheet formation

- 2D MHD high-resolution turbulence simulations [Ng et al., 2003]
- Intense current "sheets"
- Local anisotropy along local $\mathbf{B}$
Conclusion

- Parker's model for solar corona heating is realizable when an equilibrium becomes unstable.
- For RMHD and periodic boundary condition, there is only one smooth equilibrium for each mapping.
- Formation of current layers and trend toward non-equilibrium seen in numerical simulations.
- Possible topology of current sheets proposed and seen in simulation of 3D coalescence instability.
- 3D magnetic reconnection possible at a current sheet, without magnetic null points or neutral lines.
- A lot more works to be done.