Random Walk on the Surface of the Sun

Chung-Sang Ng

Geophysical Institute, University of Alaska Fairbanks

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• Amitava Bhattacharjee, UNH
• Liwei Lin, UNH
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Outline

• Introduction to the heating problem of the solar corona.
• Parker’s model for the heating of solar corona based on formation of current sheets.
• Some results from numerical simulations.
• Theoretical understandings based on random walk.
• Conclusions.
Coronium?

Unusual emission lines (first observed in 1869 eclipse), e.g., a 530.3 nm green line --- new element (Coronium)?

In fact, those are from highly ionized ions (Fe XIV for the green line) [Grotrian (1939) and Edlen (1942)]

Corona must be very hot!

X-ray emission

X-ray emission from the sun is observed since 1950’s.

Nowaday X-ray/EUV images are important tools in observations of the corona.

Hotter object emits radiation in shorter wavelength.
Wien's displacement law

“White hot” is hotter “red hot”.

\[ \lambda_{\text{max}} = \frac{b}{T} \]

\( b \sim 2.9 \times 10^{-3} \text{ mK} \) (Wien’s displacement constant) --- for black body radiation.

Light from the sun has a spectrum peaks \( \sim 500 \text{ nm} \) ==> solar surface \( \sim 6000 \text{ K} \)

A spectrum peaks \( \sim 3 \text{ nm} \) (X-ray) ==> \( T \sim 10^6 \text{ K} \)

Solar corona: heating problem

The Big Questions

The Coronal Heating Process

The Sun's outer atmosphere (the Corona) is hotter than 1,000,000°C (1,800,000°F) while the visible surface has a temperature of only about 6000°C (10,000°F). The nature of the processes that heat the corona, maintain it at these high temperatures, and accelerate the solar wind is a third great solar mystery. Usually temperatures fall as you move away from a heat source. This is true in the Sun's interior right up to the visible surface. Then, over a relatively small distance, the temperature suddenly rises to extremely high values. Several mechanisms have been suggested as the source of this heating but there is no consensus on which one, or combination, is actually responsible.

[http://science.nasa.gov/ssl/PAD/solar/quests.htm]
Solar corona: heating problem

\[ \frac{T_{\text{steam}}}{T_{\text{ice}}} \sim 1.4 \]

\[ \frac{T_{\text{corona}}}{T_{\text{photosphere}}} \sim 300 \]
## Solar corona: basic parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Photosphere</th>
<th>Corona</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temperature</td>
<td>$\sim 6 \times 10^3$ K</td>
<td>$\sim 10^6$ K</td>
</tr>
<tr>
<td>Density</td>
<td>$\sim 10^{23}$ m$^{-3}$</td>
<td>$\sim 10^{12}$ m$^{-3}$</td>
</tr>
<tr>
<td>Time scale</td>
<td>$\sim 10^4$ s</td>
<td>$\sim 20$ s</td>
</tr>
</tbody>
</table>

Magnetic fields (10~100 G) --- role in heating?

Two main theories:
- Alfvén wave (AC)
- current sheets (DC)
Magnetic field in the solar corona

Extreme UV image taken by the Transition Region and Coronal Explorer (TRACE)

[http://trace.lmsal.com/]

[Knight, 1st Ed. 2004]
Quasi equilibrium in coronal field

Extreme UV movie taken by Solar TErrestrial RElations Observatory (STEREO)

In 36 hours

Most of the time in quasi equilibrium

[http://stereo.gsfc.nasa.gov/]
Parker's model (1972) of coronal heating

2003 James Clerk Maxwell Prize to Eugene N. Parker
University of Chicago

Citation:
"For seminal contributions in plasma astrophysics, including predicting the solar wind, explaining the solar dynamo, formulating the theory of magnetic reconnection, and the instability which predicts the escape of the magnetic fields from the galaxy."
Parker's model (1972) of coronal heating

Session PR1 - Maxwell Prize Talk: Tangential Discontinuities in Untidy Magnetic Topologies.
INVITED session, Thursday morning, October 30
Kiva, ACC

[PR1.001] Tangential Discontinuities in Untidy Magnetic Topologies

Eugene N. Parker (Department of Physics, University of Chicago, Chicago, Illinois)

The tenuous corona of the Sun, and most other stars, emits X-rays from regions of million degree gas confined in re-entrant (bipolar) magnetic fields. The basic question is the heat input responsible for the million degree temperatures. There seems to be a direct association of heat input with magnetic field. We suggest that an important contribution comes from the dissipation of magnetic free energy of the generally untidy interwoven internal topology of the bipolar magnetic fields. The dissipation is provided by the trend toward surfaces of tangential discontinuity (STD’s) in the magnetic field as the field relaxes toward static equilibrium. The incipient STD’s become sites of rapid reconnection. The tendency to STD’s can be shown from the force-free equilibrium equations (for a magnetic field embedded in a gas with uniform pressure and infinite electrical conductivity). The equations have two families of complex characteristics and one family of real characteristics, viz. the field lines, giving them this curious property. The STD’s are an essential part of the equilibrium solutions because the torsion in the field is constrained to be constant along the real characteristics while the field lines wrap first one way and then the other at different locations along the field. This contradiction is resolved by the formation of STD’s. The optical analogy for the extension of field lines through an inhomogeneous field illustrates the formation of gaps in the flux surfaces, leading to the STD’s.
Heating by currents -- Ampère’s law

• plasma can carry electric current

• magnetic field is related to electric current

• current density $\mathbf{J}$ from Ampère’s law:

$$\mathbf{J} = \frac{1}{\mu_0} \left( \nabla \times \mathbf{B} - \frac{\partial \mathbf{D}}{\partial t} \right) \approx \frac{1}{\mu_0} \nabla \times \mathbf{B}$$
Heating by currents -- Ohm’s law

• Electric field \( \vec{E} = \eta \vec{J} \) (in the rest frame of the plasma, \( \eta \) is resistivity)

For \( T \sim 10^6 \text{ K} \) (\( k_B T \sim 100 \text{eV} \)), \( \eta \sim 5 \times 10^{-7} \text{ ohm-m} \)
(\( \eta \sim 7 \times 10^{-7} \text{ ohm-m} \) for stainless steel, \( 2 \times 10^{-8} \text{ ohm-m} \) for copper)

\[
\vec{E}L = \frac{\eta L}{A} (\vec{JA}) \Rightarrow \Delta V = IR
\]

• Faraday’s law:

\[
\frac{\partial \vec{B}}{\partial t} = -\nabla \times \vec{E} = -\frac{\eta}{\mu_0} \nabla \times \nabla \times \vec{B}
\]

\[
= \frac{\eta}{\mu_0} \nabla^2 \vec{B}
\]
Heating by current sheets

- Magnetic diffusion equation: \( \frac{\partial \vec{B}}{\partial t} = \frac{\eta}{\mu_0} \nabla^2 \vec{B} \)

- Magnetic diffusion time \( \tau_d = \mu_0 \frac{L^2}{\eta} \)

- For resistivity \( \eta \sim 5 \times 10^{-7} \text{ ohm}\cdot\text{m}, \quad L \sim 10^7 \text{ m} (\sim 1\% R_S), \quad \tau_d \sim 8 \times 10^6 \text{ years!} \)

- Requires \( \tau_d \sim 10^4 \text{ s} \), then \( L \sim 10^2 \text{ m} \), i.e., needs to develop small spatial scales: current sheets
Frozen-in field line condition

• In ideal MHD ($\eta = 0$), magnetic field lines are “frozen” in the fluid.
Parker's Model (1972): use $\eta = 0$

Straighten a curved magnetic loop

Photosphere

Footpoint Twisting

Smooth Uniform Field

Non equilibrium with Current Sheet
(Tangential Discontinuity)
Tangential discontinuities as current sheets

\[ \vec{J} = \nabla \times \vec{B} / \mu_0 \]

\[ I = \int \nabla \times \vec{B} \cdot d\vec{a} / \mu_0 = \oint \vec{B} \cdot d\vec{l} / \mu_0 \neq 0 \]

• Total current even for \( L \to 0 \).
Tangential discontinuities in Parker’s model

- Photospheric motion has time scale \( \tau \sim 10^4 \text{ s} \) much longer than Alfvén time.
- Corona in quasi-equilibrium --- most of the time.

Parker’s model [Astrophys. J. 1972]: No smooth force-free equilibrium exists (assume \( \eta = 0 \)), due to complex photospheric motions.

This works only when the equilibrium becomes unstable [Ng & Bhattacharjee, Phys. Plasmas 1998]
Simulations of Parker's model

- Start with a uniform $\vec{B}$ field.
- Apply random footpoint motion that twists $\vec{B}$ field.
- Current layers appear/disrupt.
- Quasi-equilibrium ($\vec{B} \cdot \nabla J = 0$) most of the time, but becomes unstable when $J$ getting large.
- Recent results from a 3D pseudo-spectral parallelized Reduced MHD (RMHD) code show that heating rate independent of $\eta$ (inverse of Lundquist number, which is very large in the solar corona).
$B_\perp$ from random footpoint motion

$z = L$

$z = 0$

$B_\perp \sim B_z \frac{l}{L} \sim B_z v_p \frac{t}{L}$
Constant footpoint motion --- exact solution

\[
\frac{B_\perp}{B_z} \sim \frac{v_L \tau_r}{L} \equiv \frac{l_r}{L} = \frac{v_L w^2}{L \eta} \gg 1
\]

where \( l_r \) is the distance a photospheric footpoint move in a resistive time \( \tau_r \sim w^2 / \eta \).

- Unphysically large \( \langle H \rangle \sim \frac{W_d}{w^2} \sim B_z^2 v_L \frac{l_r}{L} \propto \eta^{-1} \).

- A reference case for the theory and simulation.
Random walk

Step length: $l$

Location after $N$ steps: $(x,y)$

Distance after $N$ steps: $L$

Average location: $<x> = <y> = 0$

Expected distance: $<L> = N^{1/2} l$


Random walk in a random velocity field $v$:

If $l = v \tau_c$, $L$ (in time $t$) = $(t/\tau_c)^{1/2}l = v (t\tau_c)^{1/2}$, if $t > \tau_c$
**B\(\perp\) from random footpoint motion**

If dissipation is due to Ohmic heating with resistivity \(\eta\)

\[
\frac{\overline{B}_y}{B_z} \sim \frac{l_c}{L} \sim \frac{v_0}{L} \sqrt{\tau_{coh}\tau_r} \sim \frac{v_0}{L} \sqrt{\frac{\tau_{coh}w^2}{\eta}} \gg 1
\]

where \(l_c = v_0 \sqrt{\tau_r \tau_{coh}}\) is the statistically expected distance moved by a footpoint with velocity \(v_0\) in a random walk motion in a resistive time \(\tau_r \sim w^2/\eta\).

- Heating rate \(\overline{W_d} \sim \eta \int \overline{J^2} d^3 x \sim \eta B_y^2 (Lw^2)/w^2 \sim \frac{v_0^2}{L} B_z^2 \tau_{coh} w^2\)

\(\langle H \rangle \sim \frac{\overline{W_d}}{w^2}\) is independent of \(\eta\).

- If \(w \sim v_0 \tau_{coh}\), \(\langle H \rangle \sim B_z^2 v_0 w/L\), which is roughly of the same order of magnitude required for coronal heating.

- However, \(\overline{B}_y\) is unphysically large for a small \(\eta\).
Simulation of heating in tectonics model


\[ \tau_{coh} = 20 \sim 0.002 \tau_r \]

- Average heating rate almost independent of \( \eta \).
- Same heating rate even with instabilities or reconnection.

![Graph showing heating rate over time with different \( \eta \) values]
Random drive --- transverse $B$/small $\tau_{\text{coh}}$

$$\overline{B}_y(t) \equiv \left[ \frac{1}{t} \int_0^t \int \overline{B}_y^2(x, t') d^2xd't' \right]^{1/2}$$

$\tau_{\text{coh}} = 20 \sim 0.002 \tau_r$

- $\overline{B}_y$ has almost a $\eta^{-1/2}$ dependence.

- not physical $\eta^{-1/2}$ is still very small
3D Simulation of Parker’s model

- Magnetic energy limited by disruptions.

\[ \eta = \nu = 0.000625 \ (64 \times 64 \times 16) \quad \eta = 0.0003125, \ \nu = 0.000625 \ (256 \times 256 \times 32) \]
3D Simulation of Parker’s model

- Average magnetic field strength saturated in time.

\[ \eta = \nu = 0.000625 \ (64 \times 64 \times 16) \quad \eta = 0.0003125, \ \nu = 0.000625 \ (256 \times 256 \times 32) \]
3D Simulation of Parker’s model

- Energy dissipation rate saturated in time.

\[ \eta = \nu = 0.000625 \ (64 \times 64 \times 16) \quad \eta = 0.0003125, \ \nu = 0.000625 \ (256 \times 256 \times 32) \]
3D Simulation of Parker’s model

- $J_{\text{max}}$ larger for smaller $\eta$.

\[ \eta = \nu = 0.000625 \ (64\times64\times16) \quad \eta = 0.0003125, \ \nu = 0.000625 \ (256\times256\times32) \]
• Formation of thin current layers.

\[ \eta = \nu = 0.00125 \]  
\[(128 \times 128 \times 32)\]
Formation of thin current layers.

\[ \eta = 0.0003125, \quad \nu = 0.000625 \]

(256x256x32)
Random drive in 3D RMHD

- Average energy dissipation rate saturated in small $\eta$.

Longcope & Sudan (1994):

\[
\left\langle P_F \right\rangle \sim \nu_F \bar{B}_\perp \propto \eta^{-1/3}
\]

\[
\bar{B}_\perp \sim \left\{ l_F (N_{\tau_E})^{-1} \Delta^{-1/2} \right\}^{2/3} \eta^{-1/3}
\]
Random drive in 3D RMHD

- Average magnetic field strength saturated in small $\eta$.

Note that $B_z = 1$.

Longcope & Sudan (1994):

$$\bar{B}_\perp \sim \left( l_F (N_F \tau_E)^{-1} \Delta^{-1/2} \right)^{2/3} \eta^{-1/3}$$
“Slow” Sweet-Parker reconnection

- Slow reconnection rate: \( U_{\text{in}} = 2V_A / S^{1/2} \quad (S = \mu_0 V_A L / \eta) \)

- Thin and long current sheet: \( \Delta = 2L / S^{1/2} \)

From [Gurnett and Bhattacharjee, 2005]
Scaling analysis in 3D

Sweet-Parker reconnection: \( \frac{\delta}{\Delta} \sim S_{\perp}^{-1/2} \quad S_{\perp} \equiv \bar{B}_{\perp} w/\eta \)

Heating rate: \[ \bar{W} \sim \eta N \Delta L \frac{\bar{B}_{\perp}^2}{\delta} \sim \frac{\bar{B}_{\perp}^2 L L_{\perp}^2}{\tau_E} \]

If \( \tau_E < \tau_c \), no random walk:
\[ \bar{B}_{\perp} \sim B_z \frac{v_p \tau_E}{L} \sim \left[ \left( \frac{B_z v_p}{L N} \right)^2 \frac{L_{\perp}^4}{w \eta} \right]^{1/3} \]
\[ \bar{W} \sim \left( \frac{L_{\perp}^{10} B_z^5 v_p^5}{L^2 N^2 w \eta} \right)^{1/3} \]

If \( \tau_E > \tau_c \), random walk:
\[ \bar{B}_{\perp} \sim B_z \frac{v_p (\tau_c \tau_E)^{1/2}}{L} \sim \left[ \left( \frac{B_z v_p L_{\perp}}{L} \right)^4 \frac{\tau_c^2}{N^2 w \eta} \right]^{1/5} \]
\[ \bar{W} \sim \frac{L_{\perp}^2}{L} B_z^2 v_p^2 \tau_c \]

Substituting numerical parameters shows that transition at around \( \eta = 10^{-3} \)
B⊥ from random footpoint motion

If dissipation is due instability/reconnection when \( B_y \sim f B_z \)

\[
\frac{B_y}{B_z} \sim f \sim \frac{v_0}{L} \sqrt{\tau_{\text{coh}} \tau_E} \Rightarrow \tau_E \sim \left( \frac{fL/v_0}{L} \right)^2 / \tau_{\text{coh}}
\]

- Heating rate \( \bar{W}_d \sim B_y^2 (Lw^2) / \tau_E \sim \frac{L}{v_0^2} B_z^2 \tau_{\text{coh}} w^2 \)
  \( \langle H \rangle \sim \frac{\bar{W}_d}{w^2} \) is independent of \( f \) (and dissipation mechanism).

- If \( w \sim v_0 \tau_{\text{coh}} \), \( \langle H \rangle \sim B_z^2 v_0 w / L \), which is roughly of the same order of magnitude required for coronal heating.

- Now there is no unphysically large \( \bar{B}_y \), if instability and reconnection dissipates energy fast enough when \( f \sim O(1) \).
Conclusions

- Parker's model of coronal heating is studied using 2D and 3D RMHD simulations.

- Scaling laws with resistivity found in simulations can be understood by using the concept of random walk of photospheric footpoint motion.

- The saturation of the heating rate in the small $\eta$ limit seems to be robust regardless of the dissipation mechanism and how the magnetic field production is limited.

- This heating rate found in simulations and theory is at the level needed for coronal heating.