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Three-Dimensional Simulations of the Parker's Model of Solar Coronal Heating: Lundquist Number Scaling due to Random Photospheric Footpoint Motion

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Abstract. The model of Parker (1972) is one of the mostly discussed mechanisms for coronal heating and has generated much debate. We have recently obtained new scaling results in a two-dimensional (2D) version of this problem suggesting that the heating rate becomes independent of resistivity in a statistical steady state (Ng & Bhattacharjee 2008) Our numerical work has now been extended to 3D by means of large-scale numerical simulations. Random photospheric footpoint motion is applied for a time much longer than the correlation time of the motion to obtain converged average coronal heating rates. Simulations are done for different values of the Lundquist number to determine scaling. In the large Lundquist number limit, results obtained so far are consistent with the trend that the coronal heating rate is independent of the Lundquist number, as predicted by previous analysis as well as 2D simulations. In the same limit the average magnetic energy built up by the random footpoint motion tends to saturate at a constant level, due to the formation of strong current layers and subsequent disruption when the equilibrium becomes unstable.

1. Introduction

While a definitive resolution of the coronal heating problem continues to be elusive (see the recent review by Klimchuk 2006 and the monograph by Aschwanden 2005 for a comprehensive discussion), observations in recent years, especially from Yohkoh, Solar and Heliospheric Observatory (SOHO), and Transition Region and Coronal Explorer (TRACE) missions, have had a profound impact on our thinking regarding coronal heating mechanisms. The magnetic carpet, which covers the entire surface of the Sun and is constituted of magnetic fragments that are in a continual dynamical state of emergence, break-up, merging, and cancelation (Schrijver et al. 1998; Title 2000; Hagenaar 2001; Parnell 2001; Priest et al. 2002), holds a key to understanding the heating of the global corona as well as the solar wind. Approximately 90% of the magnetic flux of the quiet Sun in the network concentrations, embedded in the carpet, originates from newly emerged bipolar pairs called ephemeral regions (Martin 1988) which have a mean total value of about Maxwells. The striking images produced by TRACE appear to suggest that the corona is composed of myriads of loops of various sizes, from large to small, with footpoints rooted in the network where most of the photospheric magnetic flux resides. It is estimated that 95% of the photospheric magnetic flux closes within the magnetic carpet (or the transition region) in low-lying loops, leaving only 5% to form large-scale connections (Schrijver & Zwaan 2000).

By measuring the rate of emergence of ephemeral regions from the Michelson Doppler Imager (MDI) instrument on board SOHO, it has been found that the photospheric flux in the quiet Sun is replaced approximately every 14 hours (Hagenaar 2001). Surprisingly, however, the recycling time for magnetic flux in the solar corona is found to be only about 1.4 hours, which is about a tenth of the photospheric recycling time (Close et al. 2004). This recycling time is obtained by considering the effects of reconnection as well as the emergence and cancellation of flux (which also involve a substantial amount of reconnection). These observations suggest a far more dynamic quiet-Sun corona than previously thought, with reconnection playing a crucial role in processing the flux and releasing magnetic free energy that may heat the global corona.

Fast reconnection is mediated by the formation of thin and intense current sheets. Parker (1972, 1994) proposed that the current density in the corona is distributed generically in the form of current sheets (tangential discontinuities). He has argued that the magnetic free energy of the system will be dissipated at near-Alfvénic rates at the sites of current sheets in the presence of a very small but finite resistivity, and has attempted to demonstrate that "... the X-ray luminosity of the Sun ... is a consequence of a sea of small reconnection events —nanoflares—in the local surfaces of tangential discontinuity throughout the bipolar magnetic fields of active regions" (Parker 1994). Recently, Priest et al. (2002) have proposed an analytical model in which a hierarchy of current sheets is formed at coronal separatrix surfaces, produced by the motions of a myriad of independent but small photospheric flux elements. By analogy with geophysical plate tectonics, where the relative motion of plates under the surface of the Earth produces potentially singular dynamics above the surface, Priest et al. (2002) have described their picture a "tectonics" model.

Ng & Bhattacharjee (2008) have recently presented an analytical and numerical treatment of a simple version of the tectonics model. In these studies, following the geometry of the Parker's model (Parker 1972), we assume that closed, low-lying coronal loops which are anchored in the photosphere can be modeled in straight rectangular geometry. This initial configuration is then twisted by photospheric footpoint motion. In (Ng & Bhattacharjee 2008), this motion was restricted to depend on only one coordinate transverse to the initial magnetic field. This strong assumption has the consequence that it enables us to describe the entire dynamics by a simple set of differential equations which are easily amenable to analytical and numerical solutions for prescribed footpoint motions. The geometric constraints imposed by our assumption precludes the occurrence of reconnection and secondary instabilities, but enables us to follow for long times the dissipation of energy due to the effects of resistivity and viscosity. From these 2D simulations and in the limit of small resistivity, we found that the heating becomes independent of the resistivity of the plasma. Furthermore, we obtained scaling relations that suggest that even if reconnection and secondary instabilities were to limit the build-up of magnetic energy in such a model, the overall heating rate will still be independent of the resistivity.

Due to the initial success in our studies in 2D, we have started to extend our study to 3D using a reduced MHD (RMHD) model. Our simulation is set up to be very similar to that used by (Longcope & Sudan 1994), only that we will extend the simulations to even lower resistivity and try to confirm scaling in the large Lundquist number limit. This turns out to be more difficult than one might have expected. This is due to the fact that higher resolution runs require much more CPU time per time step, as well as the need of smaller time step for numerical stability. At the same time, average values with good statistics require a long running time (up to hundreds or thousands of Alfvén time. We will present some of our preliminary 3D results in Section 3. Before we do that, we briefly describe the setup of our simulations and some 2D results in Section 2..

2. Numerical Setup and 2D Results

We assume that the coronal plasma is sufficiently low-beta so that the dynamics can be described by the RMHD equations, as derived in Strauss (1976) under such condition, which can be written in dimensionless form as

$$\partial_t \Omega + [\phi, \Omega] = \partial_z J + [A, J] + \nu \nabla_\perp^2 \Omega, \tag{1}$$

$$\partial_t A + [\phi, A] = \partial_z \phi + \eta \nabla_\perp^2 A, \tag{2}$$

where A is the flux function so that the magnetic field is expressed as $\mathbf{b} = \hat{\mathbf{z}} + \nabla_\perp A \times \hat{\mathbf{z}}$, ϕ is the stream function so that the velocity field is expressed as $\mathbf{u} = \nabla_\perp \phi \times \hat{\mathbf{z}}$, $\Omega = -\nabla_\perp^2 \phi$ is the z-component of the vorticity, $J = -\nabla_\perp^2 A$ is the z-component of the current density, and $[\phi, A] = \partial_y \phi \partial_x A - \partial_y A \partial_x \phi$. The normalized viscosity ν is the inverse of the Reynolds number $\mathsf{R_v}$, and resistivity η is the inverse of the Lundquist number S. The normalization adopted in equations (1) and (2) is such that magnetic field is in the unit of B_z (assumed to be a constant in RMHD), velocity is in the unit of $v_A = B_z/(4\pi\rho)^{1/2}$ where ρ is a constant density, length is in the unit of the transverse length scale l, the unit of time is l/v_A , η is in the unit of $4\pi v_A l/c^2$, and ν is in the unit of $\rho v_A l$.

In Parker's model, a solar coronal loop is treated as a straight ideal plasma column, bounded by two perfectly conducting end-plates representing the photosphere. The footpoints of the magnetic field in the photosphere are frozen (line-tied). Initially, there is a uniform magnetic field along the z direction. The footpoints on the plates $z = 0$ and $z = L$ are subjected to slow, random motions $\phi(z = 0, t)$ and $\phi(z = L, t)$ that deform the magnetic field. Note that in Rappazzo et al. (2007), the boundary flow used is steady in time, unlike the random flow used in this study. The footpoint motions are assumed to take place on a time scale much longer than the characteristic time for Alfvén wave propagation between $z = 0$ and $z = L$, so that the plasma can be assumed to be in quasi static equilibrium nearly everywhere, if such equilibrium exists, during this random evolution. For a given equilibrium, a footpoint mapping can be defined by following field lines from one plate to the other. Since the plasma is assumed to obey the ideal MHD equations, the magnetic field lines are frozen in the plasma and cannot be broken during the twisting process. Therefore, the footpoint mapping must be continuous for smooth footpoint motion. Parker (1972) claimed that if a sequence of random footpoint motions renders the mapping sufficiently complicated, there will be no smooth equilibrium for the plasma to relax to, and tangential discontinuities (or current sheets) of the magnetic field must develop. Although Parker's claim has stimulated considerable debate, e.g., (van Ballegooijen 1985; Longcope & Strauss 1994), we have shown that it is valid if the equilibrium becomes unstable because there is only one smooth equilibrium for a given footpoint mapping (Ng & Bhattacharjee 1998) under RMHD and using periodic boundary conditions in transverse coordinates.

Ordinarily, the problem of calculating time-dependent solutions of equations (1) and (2) in line-tied magnetic field geometry involves all three spatial coordinates and time. As a first step (Ng & Bhattacharjee 2008), we make a strong assumption that in addition to time t and the coordinate z along which the magnetic field is line-tied, the dynamics depends on only one transverse coordinate x. Then the nonlinear terms (those involve Poisson brackets) in RMHD equations (1) and (2) become identically zero.

We have developed computer simulation codes that integrate equations (1) and (2) numerically for arbitrary footpoint displacements in both 2D and 3D. We use spectral decomposition in x and y and a leapfrog finite difference method in z. For the 2D version, we can use an implicit method for time-integration so that we can take larger time steps than is allowed by the Courant–Friedrichs–Levy condition for numerical stability of explicit methods, unlike in the 3D version. This enables us to integrate efficiently for long periods of time in order to obtain good statistics. A spatial resolution of 256×128 is used here.

In Ng & Bhattacharjee (2008), our code was first checked for the case with steady footpoint motion, i.e., with time independent $\phi(z = 0)$ and $\phi(z = L)$, since the solution can be obtained analytically. However, this is an nonphysical case since the heating rate is extremely strong, with a scaling inversely proportional to η. This is the reason why we cannot use steady state boundary flow in our study, in contrast with what is used in Rappazzo et al. (2007).

When random boundary flow $\mathbf{u}_p$ is used, we have obtained scaling laws based on long time simulations that the heating rate is independent of η in the limit of small η, i.e., when the coherence time $\tau_{\rm coh}$ (defined so that $\mathbf{u}_p(t_0)$ is uncorrelated with $\mathbf{u}_p(t)$ when $|t - t_0| > \tau_{\rm coh}$) is much smaller than the resistive time $\tau_r = w^2/\eta$, where w is a typical transverse length scaling in the boundary flow. In Fig. 1(a), the time-averaged energy dissipation rate $\bar{W}_d$ is plotted as a function of time for different levels of resistivity η. We see that the asymptotic values of $\bar{W}_d$ depend insensitively on η when η changes by a factor of 4.

However, this case is still an unphysical case due to the fact that there is no nonlinear terms to induce instabilities or magnetic reconnection to limit the growth of magnetic energy. Therefore, it is also found that the average transverse magnetic field $\bar{B}_y$ scales like $\eta^{-1/2}$, as shown in Fig. 1(b). This means that $\bar{B}_y$ will grow to a vary large value, much larger than the constant B_z field, in the limit of small η.

Ng & Bhattacharjee (2008) have also presented basic theoretical considerations that confirm these scaling results for these 2D simulations. Moreover, the same analysis can be generalized to the case in which the growth of $\bar{B}_y$ is limited by some nonlinear effects. This means that we expect we should recover the scaling law with heating rate independent of η, while the transverse magnetic field is limited to physically acceptable values.

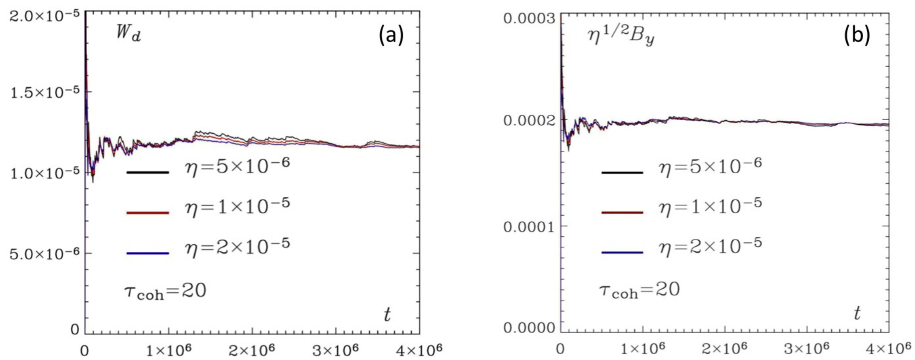

Figure 1. (a) The time-averaged energy dissipation rate $\bar{W}_d$ as a function of time for different levels of resistivity η. (b) Root-mean-square $\bar{B}_y$ multiplied by $\eta^{1/2}$ as a function of time for the same set of runs as shown in (a).

Therefore, we have started running our simulations in 3D using random boundary flows, as presented in the next Section.

3. Three-Dimensional Simulations

We have performed a series of simulations using our 3D RMHD code using a range of η to study scaling laws. The range of η has been extended to lower values for about an order of magnitude as compared with the study in Longcope & Sudan (1994) which stopped at $\eta \sim 10^{-3}$, with $\tau coh = 10 \ll \tau_r$. This extension of course requires significant increase in resolution, with our highest resolution case at $512^2 \times 64$ so far, as compared with $48^2 \times 10$ in Longcope & Sudan (1994). The main difficulty in performing these simulations is the requirement to run up to hundred or even thousands of Alfvén times in order to obtain good statistics of the average quantities under the driving of random boundary flow. Therefore, we are still continually running the cases with highest resolutions to test for convergence in time. Fig. 2 shows the preliminary scaling results we obtained so far. Nevertheless, we can see a significant deviation from the scalings obtained in Longcope & Sudan (1994) beyond $\eta \sim 10^{-3}$. Both the average heating rate in (a) and average transverse magnetic field strength in (b) begin to level off at smaller η, deviating from a supposedly power law dependence at higher η. This is consistent with our 2D simulations, as well as the scaling analysis showing that the heating rate should become independent of η at the small η limit, even when the growth of magnetic energy is limited by instabilities or reconnection. This result, if confirmed by even more comprehensive studies, is of course very important since the Lundquist number in the solar corona is so high and so it is most likely that we need to have a mechanism to provide coronal heating that is independent of the Lundquist number in order to get physically reasonable values. This research is on going and we will present more details of our latest results in a longer publication later.

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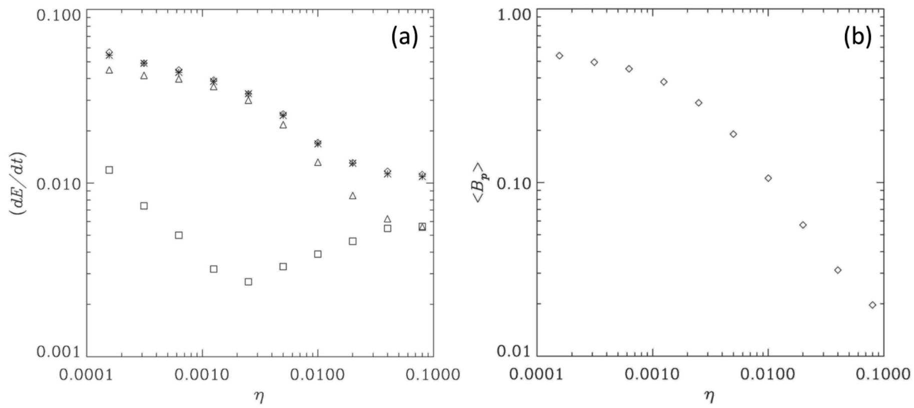

Figure 2. (a) Average energy dissipation rate for different values of η. $\triangle$ is Ohmic dissipation, $\square$ is viscous dissipation, $*$ is the total of the two, and $\diamond$ is the footpoint Poynting flux. (b) Average perpendicular magnetic field strength for different values of η.

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