Anisotropic MHD Turbulence in the Interstellar Medium and Solar Wind

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Outline

- Introduction
  - Observations in ISM and solar wind
  - Review of Hydrodynamic (HD) and Magnetohydrodynamic (MHD) turbulence
- Anisotropy due to magnetic field
- Global anisotropy: weak MHD turbulence
- Local anisotropy: 2D MHD turbulence
- Electron MHD turbulence
- Conclusion
Interstellar turbulence

Observation: power law relation between electron density spectrum and spatial scales

Kolmogorov law

From Cordes (1999)
Solar wind turbulence

Observation: power law in magnetic energy spectrum

From Goldstein & Roberts (1999)
Non-Kolmogorov observation

Interstellar media observation of the scatter-broadened image of Cygnus X-3

\[ \alpha = 1.93 \pm 0.05 \]

From Wilkinson et al. (1994)
Non-Kolmogorov observation

In situ solar wind observation

From Burlaga & Goldstein (1984); Burlaga & Mish (1986); Roberts & Goldstein (1987)
Hydrodynamic turbulence

Kolmogorov (1941): can get energy spectrum by dimensional analysis

Assumptions:
- isotropy
- local interaction in $k$-space

(energy moves from one $k$-shell to the next)

Energy cascade rate: $\varepsilon(k) \propto k^\alpha E(k)^\beta = \text{constant}$

Energy spectrum: $\int E(k) dk = \text{total energy}$

$\Rightarrow \alpha = 5/2, \quad \beta = 3/2$

Kolmogorov spectrum: $E(k) \sim C_K \varepsilon^{2/3} k^{-5/3}$

Kolmogorov constant: $C_K \sim 1.4 - 2$
MHD turbulence

Additional dependence on the Alfvén speed $V_A$

Energy cascade rate: $\varepsilon(k) \propto k^\alpha E(k)^\beta V_A^\gamma = \text{constant}$

$\Rightarrow \alpha = (5 - \gamma) / 2, \quad \beta = (3 - \gamma) / 2$

Spectral index: $\nu = \alpha / \beta = (5 - \gamma) / (3 - \gamma)$

$\gamma = 0$: Kolmogorov spectrum $E(k) \sim C_K \varepsilon^{2/3} k^{-5/3}$

$\gamma = -1$: IK spectrum $E(k) \sim C_{IK} \varepsilon^{1/2} V_A^{1/2} k^{-3/2}$

Iroshnikov (1963), Kraichnan (1965) $C_{IK} \sim 1.8 - 2.2$

Dimensionless parameter: $\chi \equiv k^{1/2} E_k^{1/2} V_A^{-1} = v_k / V_A \ll 1$
Inhibition of energy cascade

Alfvén effect: cascade develops only if two Alfvén wave packets propagating in opposite directions collide

- Weak turbulence: $\chi < 1$
- Time scales:
  - Eddy turn-over time: $\tau_N \sim 1/k\nu_k$
  - Alfvén time: $\tau_A \sim 1/kV_A = \chi\tau_N \ll \tau_N$
  - Energy cascade time: $\tau_E \sim \tau_A/\chi^2 = \tau_N/\chi >> \tau_N$

Kolmogorov cascade rate: $\varepsilon_K \sim v_k^2/\tau_N \sim k^{5/2}E_k^{3/2}$

IK cascade rate: $\varepsilon_{IK} \sim v_k^2/\tau_E \sim \varepsilon_K\chi \sim k^3E_k^2V_A^{-1} \ll \varepsilon_K$
Critique of IK theory

Anisotropy due to magnetic field

- Energy spectrum $E(k_\parallel, k_\perp)$ relative to a background $B_0$
- Energy cascade faster in $k_\perp$ than $k_\parallel$
- Alfvén time $\tau_A \sim 1/k_\parallel V_A$, $\chi \equiv k_\perp \nu_k / k_\parallel V_A$
- Critical balance cascade $\chi \sim 1$ (Goldreich & Sridhar 1995)

  recover Kolmogorov spectrum: $E(k_\perp) \sim \varepsilon^{2/3} k_\perp^{-5/3}$

  scale-dependent anisotropy: $k_\parallel \sim k_\perp^{2/3} L^{-1/3}$
Global anisotropy

New method: direct study of wave packets collision along large scale magnetic field $B_0$

3D RMHD simulation:
initial condition

$B = B_0 + V_{\perp} A \times \hat{z}$
$v = \nabla_{\perp} \phi \times \hat{z}$

(Ng & Bhattacharjee 1996)
Wave packets collision
Analytical expression for three-wave interactions confirmed with simulation.

3D RMHD simulation: three-wave interaction
c.f. analytical expressions:
+: magnetic potential $A$
×: stream function $\phi$
Weak turbulence

Keep only three-wave interaction.
Possible if $\chi \sim k_\perp v_k / k_\parallel V_A \ll 1$

- Anisotropic energy spectrum: $\text{energy} = \int E(k_\perp) k_\parallel dk_\perp$
- No parallel energy cascade ($k_\parallel$ as a parameter)
- Energy cascade rate: $\epsilon \sim v_k^2 / \tau_E \sim v_k^2 \tau_A / \tau_N \sim k_\perp^4 k_\parallel E^2(k_\perp) / V_A$
- Energy spectrum: $E(k_\perp) \propto \epsilon^{1/2} k_\perp^{-2}$
Random collision

Energy spectrum settles after a series of random collision

Keep only three-wave interactions based on analytic expression.

\[ E(k_\perp) \propto k_\perp^{-2} \] found.

Same results from kinetic equation (Galtier et al. 2000)

(Bhattacharjee & Ng 2001)
Observations of $k^{-2}$ spectra

ISM observation of the scatter-broadened image of 2005+403

From Mutel, Molnar & Spangler (1999)
Observations of $k^{-2}$ spectra

In situ solar wind observation

From Burlaga & Goldstein (1984); Burlaga & Mish (1986); Roberts & Goldstein (1987)
Local anisotropy

- simulations of 2D MHD turbulence
- no $B_0$
- decaying
- periodic in $x,y$
- $2048^2$
Globally isotropic 2D energy spectrum (logarithmic contours)
Current sheet formation

- intense current “sheets”
- along local $\mathbf{B}$
- local anisotropy
Local anisotropy

♦ structure function method: (Cho & Vishniac 2000, Maron & Goldreich 2000)

\[ F_2^U(R, Z) \equiv \left\langle \left| U(x + r) - U(x) \right|^2 \right\rangle \]

where \( Z \equiv |r \cdot \hat{B}_0 |, \ R \equiv |r \times \hat{B}_0 | \)

♦ relation between \( R, Z \): \( F_2^U(0, Z) = F_2^U(R, 0) \)

♦ \( k_{\parallel} \sim 1/ Z, \ k_{\perp} \sim 1/ R \)
Local anisotropy

- same as in critical balance turbulence
  \[ k_Z \sim k_R^{2/3} L^{-1/3} \]
  (Goldreich & Sridhar 1995)

- spectrum not Kolmogorov

\[ \hat{B}_0 \]
$k^{-3/2}$ energy spectrum

- consistent with IK spectrum
$k^{-3/2}$ energy spectra

- overlapping spectra over a period of large scale Alfvén time
Turbulence tends to be weak

- \( k_\perp v_k / k_\parallel V_A \sim 1 \)
- \( k_\perp v_{k\perp} / k_\parallel V_A < 1 \)
- no shear-Alfvén (\( v_{k\parallel} = 0 \)) wave in 2D
- \( v_{k\perp} \sim \frac{k_\parallel}{k_\perp} v_{k\parallel} \ll v_k \) for pseudo-Alfvén wave
Energy cascade rate

Fit numerical values of \( \varepsilon = C_K \varepsilon_K = C_{IK} \varepsilon_{IK} \) where

\[
\varepsilon_{IK} = k^3 E_k^2 V_A^{-1}
\]
by IK theory

\[
\varepsilon_K = k^{5/2} E_k^{3/2}
\]
by Kolmogorov theory

More consistent with IK theory
c.f. usually found values: \( C_K \sim 1.4 - 2, C_{IK} \sim 1.8 - 2.2 \)
Electron MHD

- Describes collisionless physics

  generalized Ohm's law: \[ \mathbf{E} + \frac{\mathbf{v} \times \mathbf{B}}{c} = \eta \mathbf{J} + \frac{4\pi}{\omega_{pe}} \frac{\mathbf{J}}{\mathbf{c}} - \frac{\nabla p}{ne} + \frac{\mathbf{J} \times \mathbf{B}}{nec} \]

- Faster time scale \( (\Omega_i^{-1} \sim \Omega_e^{-1}) \)

- New spatial scale \( d_e = c / \omega_{pe} \) (electron skin depth)

- Two cases: \( kd_e > 1, \ kd_e < 1 \)

- Whistler wave \( \omega = B_0 k k_\parallel / (1 + d_e^2 k^2) = \tau_W^{-1} \)

  \[ \phi_k = \pm k A_k \] (c.f. \( \phi_k = \pm A_k \) in Alfvén wave)

- Study decaying turbulence with 2D simulations
Self-interaction

Co-moving whistler wave packets also produce small scales (unlike Alfvén wave packets in MHD)

Colliding

Co-moving

Contours of current density $J$

$B_0$
EMHD turbulence

- $k^{-5/3}, k^{-7/3}$ energy spectra found for the two cases

(c.f. Biskamp et al. 1999)
Local anisotropy

- isotropic for $kd_e > 1$, anisotropic for $kd_e < 1$

\[ k_{||} \propto k_{\perp}^{3/4} \]
Effect of local anisotropy

\[ \chi = \frac{\tau_W}{\tau_N} > 1 \rightarrow \text{strong for } kd_e > 1 \]

\[ \chi \text{ closer to 1 with anisotropy for } kd_e < 1 \]

Kolmogorov-type derivations work for both cases.
Conclusion

• Wave packet interaction: a new way to study MHD turbulence with magnetic field and anisotropy.

• Weak MHD turbulence with a strong magnetic field has anisotropic spectrum $E(k_\perp) \propto k_\perp^{-2}$

• 2D MHD turbulence is found to be weak despite local anisotropy with IK spectrum $E(k) \propto k^{-3/2}$

• 2D EMHD turbulence is found to have Kolmogorov-type spectra for all $kd_e$ values partially due to local anisotropy

• Future works: compressibility, collisionless effects