Four-Field Equations: a New Model for Weakly Compressible MHD Turbulence in the Solar Wind and the Interstellar Medium

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Turbulent plasmas in the solar wind and the interstellar medium often contain a large directed magnetic field, and the plasma beta ($\beta$) is frequently of order unity. To describe the turbulent dynamics of these plasmas, we have derived a system of reduced equations from the fully compressible MHD equations using the Mach number of the turbulent velocity field (in the plasma frame) as a small parameter. These reduced equations constitute a closed system and involve four field variables: the magnetic flux, parallel vorticity, pressure and parallel flow. If $\beta \ll 1$, these equations reduce to the Rosenbluth-Strauss equations or the nearly incompressible MHD (NI-MHD) equations of Zank and Matthaeus. However, for $\beta \sim 1$, if the background equilibria have spatial inhomogeneities, the effect of compressibility is shown to enter at leading order. It cannot thus be claimed that density fluctuations are enslaved to an incompressible flow field, and that velocity and magnetic field fluctuations are independent of compressibility. Numerical simulation shows that if the background equilibria have even weak spatial inhomogeneities, the predictions of the four-field model differ significantly from NI-MHD. In such cases, the r.m.s pressure fluctuations can be of the order of the Mach-number, and the parallel magnetic field can attain energy equipartition with the velocity and pressure fluctuations. The predictions of the four-field equations are compared with fully compressible MHD simulations, and found to be in agreement. The scaling of density fluctuations with Mach number are compared with solar wind data from Helios 1 and 2, and found to account qualitatively for the scaling range observed. Further observational tests of the four-field model are suggested.