Turbulent Magnetic Reconnection Near a 3D Magnetic Null

C. S. Ng

Space Science Center
Institute for the Study of Earth, Oceans, and Space
University of New Hampshire, Durham, NH 03824
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C. S. Ng, chung-sang.ng@unh.edu, Space Science Center, Institute for the Study of Earth, Oceans, and Space, University of New Hampshire, Durham, NH 03824

Magnetic reconnection is considered near a three dimensional magnetic null point with the presence of small-scale magnetic field fluctuations. There has been an ongoing controversy about the role and importance of a magnetic null point in reconnection. While it can be argued that a region of smooth magnetic field without null/closed field line will evolve ideally, it is not totally clear whether the flow around a null must be singular. This is because a smooth field line velocity flow can always be found locally around a null point if the determinant of the magnetic field gradient tensor at the null point is nonzero. One possible way for nonideal field line flow is to consider time dependent field, especially with small-scale fluctuations, which are of small strength compare with the typical large-scale magnetic field but may have comparatively strong magnetic gradients locally at some points in space or time. This can potentially change the determinant of the magnetic field gradient tensor to zero at some points in time and thus allow nonideal field line flow. We test this idea with numerical experiments to see if small-scale fluctuations can indeed change the magnetic field topology around a null point and thus can allow magnetic reconnection. Possible applications of this idea to space environments will be discussed. (This work is support by NSF grant AST-0434322.)
Outline

◊ Magnetic nulls are important in many theories of magnetic reconnection

◊ This importance has been questioned recently [Boozer 2002], since ideal flow can be found near an isolated null point.

◊ An often forgotten fact: there are almost always small-scale fluctuations at and around a macroscopic magnetic null point that changes its topological structure.

◊ This physics is demonstrated with numerical experiments.

◊ New null points (sometimes in a large number) can form with fluctuations.

◊ The proof of ideal flow around an isolated null does not go through if there is another nearby null point.

◊ A clear-cut separatrix cannot be identified for a system with fluctuating null points.
2D Reconnection at X-Points

In most 2D models, magnetic reconnection happens around X-points ($B_\perp = 0$), e. g., [Parker, 1963; Petschek, 1964].

[adopted from Lau & Finn, 1990]
3D Reconnection at Magnetic Null Points

In 3D, a truly 2D X-point is actually a line (neutral line or closed field line), which is just a special case. In general, it changes into 3D magnetic null points ($\mathbf{B} = 0$).

[adopted from Lau & Finn, 1990]
Importance of Magnetic Nulls in Reconnection

Many authors have considered the importance of magnetic null points in reconnection, e.g., [Dungey, 1963; Stern, 1973; Cowley 1973, Vasyliunas 1984; Greene, 1988; Lau & Finn 1990].

◊ The fan surface of a null point is a separatrix (separating two regions of magnetic fields) when the plasma motion is ideal:

\[
\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B})
\]

i.e., when magnetic field lines are “frozen” in the plasma.

◊ Plasma flow across a separatrix must then be non-ideal and reconnection of magnetic field lines must happen.

◊ A null is where magnetic fields with different directions meet.
**Generalized Magnetic Field Line Velocity**

Without specifying the properties of the medium, the magnetic field is evolving generally through the Faraday equation:

\[
\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}
\]

For a given \( \mathbf{B} \) with a certain topology, if one can find a scalar field \( \Phi(\mathbf{x}) \) s.t.

\[
\mathbf{E} = -\mathbf{u} \times \mathbf{B} - \nabla \Phi
\]

then the magnetic field is “frozen” in the generalized magnetic field line flow, with velocity \( \mathbf{u} \), where

\[
\mathbf{u}_\perp = \left( \mathbf{E} + \nabla \Phi \right) \times \mathbf{B} / B^2
\]

such that

\[
\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B})
\]

One way to see the possible importance of a magnetic null in reconnection is to note that \( \mathbf{u}_\perp \) may be singular at \( B = 0 \), if \( \mathbf{E} + \nabla \Phi \neq 0 \).
Solving for $\Phi(x)$

$\Phi(x)$ can be solved by

$$B \cdot \nabla \Phi = -B \cdot E$$

or

$$\frac{\partial \Phi}{\partial x_\parallel} = -\hat{x}_\parallel \cdot E$$

where $\hat{x}_\parallel = \hat{B}$ or $-\hat{B}$ and $x_\parallel$ is the distance along $\hat{x}_\parallel$.

◊ $\Phi(x)$ has to be solved by specifying “boundary conditions”, i.e., the values of $\Phi(x_0)$ at some points $x_0$.

◊ In general, $\Phi(x)$ cannot be set at two points that are connecting by a magnetic field line.

◊ More than one solution can be found by choosing different $\Phi(x_0)$ or by using different set of $x_0$.

◊ Reconnection happens when one cannot solve for a continuous and single-valued $\Phi(x)$, or when $u_\perp = \infty$. 
A case when $\Phi(x)$ can always be solved
Within a region where $B$ is continuous and is having topologically the same structure of a uniform magnetic field, then $\Phi(x)$ can always be solved by specifying $\Phi(x_0)$ and integrating along the field lines.
Closed Field Line: $\Phi(x)$ cannot be solved generally

When there is a closed field line (either a field line close on itself or two field lines having the same start and end points), a loop integral along the closed line is usually nonzero for a general $E$ with $\nabla \times E \neq 0$. So,

$$\Phi(x_0) = \Phi(x_0) - \oint dx_{||} \cdot E = \Phi(x_0) - \int da \cdot \nabla \times E$$

$$\neq \Phi(x_0)$$

i.e., $\Phi(x)$ is not single-valued.
Solving $u$ Around a Null [Boozer 2002]

If there is a continuous field line flow around a null s.t.
\[
\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B}) = -\nabla \times \mathbf{E}
\]
at the null point $\mathbf{x} = 0$, we have
\[
\mathbf{u}_0 \cdot \nabla \mathbf{B} \bigg|_0 = \nabla \times \mathbf{E}_0
\]
So $\mathbf{u}_0$ can be solved if $\nabla \mathbf{B} \bigg|_0 \neq 0$, i.e., if all its eigenvalues are nonzero. A finite $\mathbf{u}_\perp$ around the null point is then calculated by
\[
\mathbf{u}_\perp = (\mathbf{E} + \nabla \Phi) \times \mathbf{B} / B^2
\]
requiring that $(\mathbf{E} + \nabla \Phi)_0 = 0$, s.t.
\[
\mathbf{E} + \nabla \Phi = \nabla (\mathbf{E} + \nabla \Phi) \bigg|_0 \cdot \mathbf{x} = O(|\mathbf{x}|)
\]
near the null point.

So it is argued that reconnection does not have to happen at a magnetic null point.
Some Remarks on Boozer’s Argument

◊ As pointed out before, more than one solution for \( u \) can be solved by choosing different boundary conditions. While choosing a general \( \Phi(x_0) \) at some outer boundaries will most likely result in a discontinuous \( \Phi \) at the null point, since magnetic field lines passing through different regions converge at the null, it is possible to choose a continuous \( \Phi(x_0) \) near the null instead. However, discontinuities in \( \Phi \) may then appear in the outer regions.

◊ A physical system usually have more than one (macroscopic) null points. Then the consideration around an isolated null may not apply. A null-null line can also play an important role [Lau & Finn 1990].

◊ A macroscopic null (in the geophysical or astrophysical sense) is in general a network of microscopic nulls, due to the ever-present small-scale fluctuations. Therefore it is seldom an isolated null even in a region close to it.
Macroscopic Null with Small-Scale Fluctuations

The effect of small-scale fluctuations near a null is a main reason why the null point is important in magnetic reconnection, e.g, [Greene 1988]. One major consequence, which has not been given much attention, of such fluctuations is that many more microscopic nulls exist (with number and topology changing in time) around the macroscopic null.

Consider

\[ \mathbf{B} = \mathbf{B}_M + \delta \mathbf{B} \approx \nabla \mathbf{B}_M \bigg|_0 \cdot \mathbf{x} + \delta \mathbf{B} \]

i.e., the macroscopic field \( \mathbf{B}_M \) has a null at \( \mathbf{x} = 0 \). It is easy to see that if the fluctuation field \( \delta \mathbf{B} \) has gradient of the order or greater than that of \( \mathbf{B}_M \), i.e.,

\[ |\nabla \delta \mathbf{B}| \geq |\nabla \mathbf{B}_M|_0 \]

there can be additional null points near \( \mathbf{x} = 0 \). This can happen even if \(|\delta \mathbf{B}|\) is small, but is of small enough scale, since the length scale of \( \mathbf{B}_M \) is usually very large.
Macroscopic Null with Small-Scale Fluctuations

Far away from $x = 0$, the macroscopic field is still that of $B_M$ with a null at $x = 0$. 
Geophysical Macroscopic Nulls

In the magnetosphere, the gradient of magnetic field near a null can be of the order

\[ |\nabla B_M| \sim 10^{-8} \text{T}/10^6 \text{m} \sim 10^{-14} \text{T/m} \]

Microscopic fluctuations can come from the combination of many effects: fluid turbulence, small-scale currents, motion of charged particles, etc. E.g., just to see that such fluctuations must exist, consider the magnetic field of an electron, \( \delta B \sim \mu_0 e v_e / 4 \pi r^2 \), where \( r \) is the distance to the electron, not to the macroscopic null, and so can be taken as small as classical physics allows. Since the electron density in the magnetosphere is of the order of 10 cm\(^{-3}\), let us choose \( r = 10^{-4} \text{ m} \) so that we do not need to consider other charged particles. So

\[ |\nabla \delta B| \sim \mu_0 e v_e / 2 \pi r^3 \]

\[ \sim \frac{13 \times 10^{-7} \text{Tms/C} \times 1.6 \times 10^{-19} \text{C} \times 10^5 \text{m/s}}{2 \pi \times (10^{-4} \text{ m})^3} \]

\[ \sim 10^{-9} \text{T/m} \gg |\nabla B_M| \]
Generating Microscopic Nulls: a Simple Model

Consider \( \mathbf{B} = \mathbf{B}_M + \delta \mathbf{B} \)

\[
\mathbf{B}_M = -\lambda \hat{x} - \mu y \hat{y} + (\lambda + \mu) z \hat{z}
\]

\[
\delta \mathbf{B} = \epsilon \left[ \hat{x} \sin ky + \hat{y} \sin kz + \hat{z} \sin kx \right]
\]

There is always a null at \( x = 0 \), the macroscopic null point. When \( \epsilon \) or \( k \) is large enough, other nulls can be found by solving

\[
(\lambda + \mu) z + \epsilon \sin \left\{ \frac{\epsilon k}{\lambda} \sin \left[ \frac{\epsilon k}{\mu} \sin kz \right] \right\} = 0
\]

then
\[
y = (\epsilon \sin kz) / \mu, \quad x = (\epsilon \sin ky) / \lambda.
\]

◊ For a fixed \( k \), only one null for small enough \( \epsilon \).

◊ For larger \( \epsilon \), A-B nulls pair begin to appear.

◊ Number of nulls can become very large.

◊ At larger distance, the global structure is still that of the original A null.

◊ Near the null point, magnetic field lines structure becomes complicated.

◊ Basic building block is an A-B-A null structure.
Generating Microscopic Nulls: a Simple Model

One null when no $\delta B$. 

\[ \begin{align*}
    k &= 4.000000 \\
    \epsilon &= 0.000000 \\
    \mu &= 0.0400000 \\
    \lambda &= 0.0600000 
\end{align*} \]
Generating Microscopic Nulls: a Simple Model

Still one null when $\delta B$ is very small, but can change into an $A_s$ null.

\[
\begin{align*}
k &= 4.000000 \\
\epsilon &= 0.00100000 \\
\mu &= 0.04000000 \\
\lambda &= 0.06000000 
\end{align*}
\]
Generating Microscopic Nulls: a Simple Model

5 nulls (note symmetry) appear for larger $\epsilon$. 

$k = 4.00000$
$\epsilon = 0.0060000$
$\mu = 0.040000$
$\lambda = 0.060000$
Generating Microscopic Nulls: a Simple Model

17 nulls.

\[ \begin{align*} k &= 4.00000 \\
\epsilon &= 0.0100000 \\
\mu &= 0.0400000 \\
\lambda &= 0.0600000 \end{align*} \]
Generating Microscopic Nulls: a Simple Model

125 nulls.

\[ k = 4.00000 \]
\[ \epsilon = 0.0200000 \]
\[ \mu = 0.0400000 \]
\[ \lambda = 0.0600000 \]
Topologically Possible A-B-A Null Structure
Type (i): new A-B nulls pair on the spine of the original A null.
Topologically Possible A-B-A Null Structure

Type (ii): new A-B nulls pair on the fan surface of the original A null.
Topologically Possible A-B-A Null Structure
Type (iii): new A-B nulls pair not on the spine or fan surface of the original A null.
A-B-A Null Structure Seen in the Model

\[
\begin{align*}
k &= 4.00000 \\
\epsilon &= 0.00600000 \\
\mu &= 0.04000000 \\
\lambda &= 0.06000000
\end{align*}
\]
Details of the New A, B Nulls
Details of the New A-B Nulls Pair

(Note the approximate trace of the null-null line)
Boozer’s Arguments Non-applicable for the System of Microscopic Nulls

◊ (a) $|\nabla B|_0 = 0$ when nulls pair appears or disappears.

◊ (b) Closed field lines in type (i) and (iii) A-B-A null structure.

◊ (c) Consider type (ii) structure

Since the proof requires $E + \nabla \Phi = \nabla (E + \nabla \Phi)_0 \cdot x$ near null A, effectively setting $\Phi$ at the points near it, e.g., $\Phi_C$, $\Phi_D$. However, these points all connect to a nearby B null through field lines so there will be multiple values of $\Phi_B$ at B.
Random Evolution of Microscopic Nulls

Consider $\mathbf{B} = \mathbf{B}_M + \delta\mathbf{B}$ with a macroscopic field $\mathbf{B}_M$ having an A null at $x = 0$ and $\delta\mathbf{B}$ generated by assigning random Fourier amplitudes that evolve in “time” with random phase.

At first, only the original A null with a clear separatrix.

\[ t = 0.000000 \]
\[ k = 4.00000 \]
\[ \epsilon = 1.50000e-05 \]
\[ \mu = 0.0400000 \]
\[ \lambda = 0.0600000 \]
Random Evolution of Microscopic Nulls

3 nulls.

Field lines coming near the original fan surface can go to either positive or negative $z$ direction.
Random Evolution of Microscopic Nulls

5 nulls.

A clear-cut separatrix cannot be identified when null points are fluctuating, appearing and disappearing.

\[
\begin{align*}
t &= 3.00000 \\
k &= 4.00000 \\
\epsilon &= 1.50000 \times 10^{-5} \\
\mu &= 0.0400000 \\
\lambda &= 0.0600000 
\end{align*}
\]
Random Evolution of Microscopic Nulls

7 nulls.
Movement of field lines becomes more rapid with more nulls appearing.

\[ t = 5.00000 \]
\[ k = 4.00000 \]
\[ \epsilon = 1.50000e-05 \]
\[ \mu = 0.0400000 \]
\[ \lambda = 0.0600000 \]
Random Evolution of Microscopic Nulls

9 nulls.

Movie at
http://plasma4.sr.unh.edu/ng/trmrtp_16-1.gif

\begin{align*}
t &= 13.7000 \\
k &= 4.00000 \\
\epsilon &= 1.50000e-05 \\
\mu &= 0.0400000 \\
\lambda &= 0.0600000
\end{align*}
Conclusions

◊ A macroscopic magnetic null can become a network of microscopic nulls when small-scale fluctuations are included.

◊ When the new null-null structure is taken into account, a recent proof showing ideal field line flow around an isolated null is not applicable. This restores the importance of null points in 3D magnetic reconnection because there must be non-ideal field line flow.

◊ For the case with time fluctuating microscopic nulls, a clear-cut separatrix cannot be identified. Field lines from the original fan surface can move across the surface in either direction.