Turbulent Magnetic Reconnection in High-Lundquist Number Two-Dimensional Resistive MHD Simulations

C. S. Ng, S. Ragunathan, and E. Yasin

Geophysical Institute, University of Alaska Fairbanks

International Cambridge Workshop on Magnetic Reconnection 2011
University of New Hampshire, Durham, New Hampshire, U.S.A., August 18, 2011
This work is supported by a NASA grant NNX08BA71G and a NSF grant AGS-0962477.
“Slow” Sweet-Parker reconnection

- Slow reconnection rate: \( U_{in} = 2V_A / S^{1/2} \) \( (S = \mu_0 V_A L / \eta) \)
- Thin and long current sheet: \( \Delta = 2L / S^{1/2} \)
- Reconnection rate might be higher in Hall MHD or kinetic theory, but is it possible to have fast reconnection within resistive MHD?

From [Gurnett and Bhattacharjee, 2005]
Recent results on fast reconnection in resistive MHD

From [Lapenta, 2008]
Recent results on fast reconnection in resistive MHD

From [Loureiro et al., 2009]
Recent results on fast reconnection in resistive MHD

From [Bhattacharjee et al., 2009]
Secondary island (plasmoid) formation in resistive MHD

FIG. 1 (color online). Contour plots of the current density showing the time evolution of an SP current sheet for $S = 10^8$. The times shown are, from top to bottom, $t = 0.20\tau_A$, $t = 0.40\tau_A$, $t = 0.45\tau_A$ and $t = 0.50\tau_A$. The domain shown is $-\delta_{SP} \leq x \leq \delta_{SP}$ (inflow direction, vertical), and $-0.12L \leq y \leq 0.12L$ (outflow direction, horizontal), where $\delta_{SP} \approx 10^{-4}$ is the SP layer width and $L = 1$ is the (half-)length of the current sheet (see text; only the central half of the simulation box is shown).

FIG. 2 (color online). Current density for $S = 10^4$, $S = 10^5$, $S = 10^6$ and $S = 10^7$. $S = 10^8$ is shown in Fig. 1.

From [Samtaney et al., 2009]
Secondary island (plasmoid) formation in resistive MHD

From [Bhattacharjee et al., 2009]
Reconnection in 2D island coalescence instability

\[ \frac{\partial \Omega}{\partial t} + [\phi, \Omega] = [A, J] + \nu \nabla_\perp^2 \Omega , \quad \frac{\partial A}{\partial t} + [\phi, A] = \eta \nabla_\perp^2 A \]

- Resistive MHD:

- Similar to the system simulated in [Bhattacharjee et al. 2009].

Figure 1. Contour plots of the initial flux function \( A \). Non-negative contours are solid and negative contours are broken. Contours levels are from -0.4 to 0.4 with an increment of 0.025.

Figure 2. Contour plots of \( A \) at \( t = 1 \) when the coalescence of the two islands has begun, using same contour levels as in Fig. 1.
Resistive 2D incompressible MHD equations

The equations to be simulated are the 2D incompressible MHD equations:

\[
\frac{\partial \Omega}{\partial t} + [\phi, \Omega] = [A, J] + \nu \nabla^2 \Omega ,
\]

\[
\frac{\partial A}{\partial t} + [\phi, A] = \eta \nabla^2 A ,
\]

where \( B = \nabla_\perp A \times \hat{z} \) is the magnetic field, \( \mathbf{v} = \nabla_\perp \phi \times \hat{z} \) is the fluid velocity, \( \Omega = -\nabla^2_\perp \phi \) is the \( z \)-component of the vorticity, \( J = -\nabla^2_\perp A \) is the \( z \)-component of the current density, \( [\phi, A] \equiv \phi_y A_x - \phi_x A_y \), \( \eta \) is the resistivity, and \( \nu \) is the viscosity. \( \eta = \nu \) is used in simulations presented here.

A few cases with \( \eta \gg \nu \) has been used that show very similar dynamics.
Boundary and initial conditions

Periodic boundary condition will be used in both $x$ and $y$ directions. To be specific, the simulation domain is $0 \leq x \leq 1, \ 0 \leq y \leq 1$.

The initial equilibrium is chosen to be:
\[ A(x, y, t = 0) = A_0 \sin(2\pi x)\sin(2\pi y), \]
with
\[ A_0 = 0.4, \]
to be specific. To start the instability, a small initial flow is added:
\[ \phi(x, y, t = 0) = \phi_0 [\cos(2\pi x) - \cos(2\pi y)], \]
with
\[ \phi_0 = 0.002, \]
which is chosen small enough such that there is a clear linear phase.
Simulation method

The pseudo spectral code used here is based on fast Fourier transform (FFT) on a 2D bi-periodic domain. It is de-aliased by the standard 2/3 rule. The nonlinear term is calculated in the physical space on a uniform grid of collocation points. A second order predictor-corrector method is used for time integration. The code is parallelized using a parallel version of the FFT. The accuracy of the results from this code has been compared with results using finite volume or spectral-element methods \[Ng \textit{et al.} 2008\]. Convergence for some lower-resolution runs has been confirmed by running with much higher resolutions, up to $2048^2$. 
High accuracy in the pseudo spectral method

The accuracy of this code is checked with respect to how well it preserves conservation laws. In this run, the fractional difference between the two sides of the following energy conservation equation is below $10^{-5}$:

$$\frac{d}{dt}(E_K + E_M) = -\nu\langle \Omega^2 \rangle - \eta\langle J^2 \rangle$$

A run with $\eta = \nu = 5 \times 10^{-5}$ using a resolution of $2048^2$.

[c.f. Ng et al., 2008]
The fractional difference between the two sides of the following conservation equation for the magnetic helicity is below $10^{-6}$:

$$\frac{d\langle A^2 \rangle}{dt} = -\eta E_M$$

A run with $\eta = \nu = 5 \times 10^{-5}$ using a resolution of $2048^2$. 

[c.f. Ng et al., 2008]
Resolution and resistivity used in resolved runs

Note that

\[ S = B_{\text{max}} \frac{L}{\eta} = 2\pi A_0 / \eta \approx 2.51 / \eta \]

Highest \( S \) so far is

\[ S_{\text{max}} \approx 2 \times 10^5 \]

using a resolution of \( 8192^2 \).
A typical time series of the reconnected flux

The reconnection rate (peak value) is the maximum of the slope of the time series.

A run with $\eta = \nu = 5 \times 10^{-5}$ using a resolution of $2048^2$. 
Reconnection flux vs. Lundquist number $S$
Reconnection rate vs. resistivity $\eta$

The reconnection rate (peak value) scales roughly with $\eta^{1/2}$, consistent with the Sweet-Parker theory.

No sign of leveling off or increasing of the reconnection rate at the small resistivity limit.
Even finer scales appears at small resistivity

At larger resistivity, the current sheet width is the finest scale of the system and it scales with $\eta^{1/2}$ as predicted by the Sweet-Parker theory.

At smaller resistivity below $\sim 10^{-4}$, fine structures thinner than the current sheet width appear and scale roughly with $\eta$ (or even thinner).
Even finer scales appears at small resistivity

\[ l_\Omega \propto \eta^{1/2} \]

The same is true for the small scales in the vorticity field.

This means that even higher linear resolution (proportional to \( \eta^{-1} \)) is needed to resolve all fine scales.
Finer structures in the reconnection out flow

The finest structures are found at the location when the reconnection out flow collides with another island.

Note also that these structures can have orientations different from the reconnection current sheet.

No secondary island has been found along the reconnection current sheet in resolved runs.

A run with $\eta = \nu = 1.25 \times 10^{-5}$ ($S \sim 2 \times 10^5$) using a resolution of $8192^2$. 
Absence of secondary island

$t = 0.00e+00, J_y [-8.22e+00, 3.04e-01]$

Our run with $\eta = \nu = 1.25 \times 10^{-5} (S \sim 2 \times 10^5)$ using a resolution of $8192^2$.

No secondary island is found in our run.
Figure 3: (a) Contour plots of $A$ (using same contour levels as in Fig. 1) showing the formation of secondary islands which are being ejected out along the reconnection outflow in a finite difference simulation using a resolution of $256^2$ for the case with $\eta = \nu = 10^{-4}$. (b) Contour plots of $A$ at a similar time as in (a) but for a pseudo spectral simulation using a resolution of $1024^2$ for the same $\eta$ and $\nu$. (c) The reconnection rates as functions of time for the case (a) (in red) and (b) (in black).

We have seen many cases of “secondary island” formation (along with increased reconnection rate) when the runs are not well resolved. No secondary island is found when resolution is sufficiently increased using the same $\eta$ and $\nu$.  

Secondary islands in under-resolved cases
Secondary islands in turbulent cases

If we add random noise throughout the simulation, secondary islands do form as shown in (a) for a case with $\eta = \nu = 3 \times 10^{-4} (S \sim 8.4 \times 10^3)$ using $256^2$, while there is no secondary island for the no-noise case in (b). The reconnection rate can spike up at times as shown in the black curve in (c) as compared with the case with no-noise (red). However, the total reconnection time for the two cases are about the same, as shown in (d).
Similarly, adding random noise for higher $S$, secondary islands also form as shown in (a) for a case with $\eta = \nu = 2 \times 10^{-4} (S \sim 1.3 \times 10^4)$ using $512^2$, while there is no secondary island for the no-noise case in (b). The reconnection rate can spike up at times as shown in the black curve in (c) as compare with the case with no-noise (red). Now, the total reconnection time for the case with noise is shorter.
Again, adding random noise for even higher $S$, secondary islands form as shown in (a) for a case with $\eta = \nu = 10^{-4}$ ($S \sim 2.5 \times 10^4$) using $1024^2$, while there is no secondary island for the no-noise case in (b). The reconnection rate spikes up even more as shown in the black curve in (c) as compare with the case with no-noise (red). Now, the total reconnection time for the case with noise is getting even shorter comparatively.
Reconnection rate in turbulent cases

\[ E.R. = 0.1 \times 10^{-5} \]

\[ T_c = 0.25 \]
Conclusions

• Numerical simulations of magnetic reconnection in the configuration of island coalescence instability have been performed systematically using a well-tested pseudo spectral 2D MHD code for a range of resistivity and resolution, with the highest Lundquist number at about $2 \times 10^5$, using a resolution of $8192^2$.

• The reconnection rate at the small resistivity limit is found to be consistent with the Sweet-Parker theory.

• No secondary island is found in all resolved runs, although the highest Lundquist number is at the similar level as in other studies that showed secondary island formation and increased reconnection rate.

• Secondary islands are found in cases with insufficient resolution. Such formation is absent when the resolution is sufficiently increased.

• Fine scales that scale roughly with $\eta$ is found in the small $\eta$ limit. This means that even higher linear resolution (proportional to $\eta^{-1}$) is needed to have resolved runs.

• Secondary islands can form in simulations when random noise is imposed. For higher $S$ cases, reconnection rate becomes higher and appears to be independent of $S$.