Energy Distribution of Nanoflares in Three-Dimensional Simulations of Coronal Heating

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Abstract

In a recent computational campaign [Ng et al., ApJ 747 109, 2012] to investigate a three-dimensional model of coronal heating using reduced magnetohydrodynamics (RMHD), we have obtained scaling results of heating rate verse Lundquist number based on a series of runs in which random photospheric motions are imposed for Hundreds to Thousands of Alfvén time in order to obtain converged statistical values. Using this collection of numerical data, we have performed additional statistical analysis related to the formation of current sheets and heating events, or nanoflares [Parker ApJ 330 474, 1988]. While there have been many observations of the energy distribution of solar flares, there have not been many results based on large-scale three-dimensional direct simulations due to obvious numerical difficulties. We will present energy distributions and other statistics based on our simulations, calculated using a method employed in [Dmitruk & Gómez, ApJ 484 L83, 1997]. We will also make comparisons of our results with observations.

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Outline

• Introduction to the Parker’s model for the heating problem of the solar corona and our simulation results.

• Parker’s nanoflare heating model vs observations

• Energy distributions of nanoflares in our simulations

• Conclusions.
Coronal Heating: Topological Dissipation

- Parker coronal heating scenario (Parker 1972)

1. Sufficiently tangled fields cannot relax to smooth equilibrium
2. Current sheets form and heat the plasma ohmically while magnetic reconnection reduces topological complexity

TRACE 171 Å Fe IX/X ~1,000,000 K
Goal of our study: Investigate how heating rate scales with dissipation coefficients as model evolves in statistical steady state.

Key challenges in computational study:
1. Separation of scales required very high resolutions
2. Need long time integrations for good statistics
Reduced Magnetohydrodynamics

- Magnetohydrodynamics (MHD):
  - Applies to electrically conducting fluids
  - Equations combine Navier-Stokes and Maxwell's Equations
- Reduced Magnetohydrodynamics (RMHD):
  - Low frequency limit of MHD, very well suited for simulations
  - Applicable in systems permeated by strong uniform magnetic field

\[
\frac{\partial \Omega}{\partial t} + [\phi, \Omega] = \frac{\partial J}{\partial z} + [A, J] + \nu \nabla^2 \Omega
\]

\[
\frac{\partial A}{\partial t} + [\phi, A] = \frac{\partial \phi}{\partial z} + \eta \nabla^2 A
\]

\[
\mathbf{B} = \hat{\mathbf{z}} + B_\perp = \hat{\mathbf{z}} + \nabla_\perp A \times \hat{\mathbf{z}} \quad \text{--- magnetic field,}
\]

\[
\mathbf{v} = \nabla_\perp \phi \times \hat{\mathbf{z}} \quad \text{--- fluid velocity,}
\]

\[
\Omega = -\nabla_\perp^2 \phi \quad \text{--- vorticity,}
\]

\[
J = -\nabla_\perp^2 A \quad \text{--- current density,}
\]

\[
\eta \quad \text{--- resistivity, } \nu \quad \text{--- viscosity,}
\]

\[
[\phi, A] \equiv \phi_y A_x - \phi_x A_y
\]
• Formation of thin current layers.

$$\eta = \nu = 0.00125$$

(128x128x32)
• Formation of thin current layers.

$\eta = 0.0003125$, $\nu = 0.000625$  
(256x256x32)
Random drive in 3D RMHD

- Average energy dissipation rate saturated in small $\eta$.

Longcope & Sudan (1994):

$$\langle P_F \rangle \sim \nu_F \bar{B}_\perp \propto \eta^{-1/3}$$

$$\bar{B}_\perp \sim \{ l_F (N \tau_E)^{-1} \Delta^{-1/2} \}^{2/3} \eta^{-1/3}$$

- Saturated level $\sim$ realistic heating rate of the corona
Random drive in 3D RMHD

- Average magnetic field strength grows slower in small $\eta$.

Note that $B_z = 1$.

Longcope & Sudan (1994):

$$\bar{B}_\perp \sim \left( l_F (N \tau_E)^{-1} \Delta^{-1/2} \right)^{2/3} \eta^{-1/3}$$

- Additional physics is needed for saturation in small $\eta$. 
Scaling analysis in 3D

Sweet-Parker reconnection: \[ \frac{\delta}{\Delta} \sim S_\perp^{-1/2} \quad S_\perp \equiv \bar{B}_\perp w/\eta \]

Heating rate: \[ \bar{W} \sim \eta N \Delta L \frac{\bar{B}_\perp^2}{\delta} \sim \frac{\bar{B}_\perp^2 LL^2_\perp}{\tau_E} \]

If \( \tau_E < \tau_c \), no random walk:
\[ \bar{B}_\perp \sim B_z \frac{v_p \tau_E}{L} \sim \left[ \left( \frac{B_z v_p}{LN} \right)^2 \frac{L^4_\perp}{w\eta} \right]^{1/3} \]
\[ \bar{W} \sim \left( \frac{L^{10}_\perp B_z^5 v_p^5}{L^2 N^2 w \eta} \right)^{1/3} \]

If \( \tau_E > \tau_c \), random walk:
\[ \bar{B}_\perp \sim B_z \frac{v_p (\tau_c \tau_E)^{1/2}}{L} \sim \left[ \left( \frac{B_z v_p L_\perp}{L} \right)^4 \frac{\tau_c^2}{N^2 w \eta} \right]^{1/5} \]
\[ \bar{W} \sim \frac{L^2_\perp}{L} B_z^2 v_p^2 \tau_c \]

Substituting numerical parameters shows that transition at around \( \eta = 10^{-3} \)

Results based on a series of runs

Accumulated simulation data can be used for statistical studies of other properties.

<table>
<thead>
<tr>
<th>Run</th>
<th>$\eta$</th>
<th>$\nu$</th>
<th>$B_\perp$</th>
<th>$S_\perp$</th>
<th>$W_\eta$</th>
<th>$W_\nu$</th>
<th>Poynting</th>
<th>$T/\tau_A$</th>
<th>Resolution</th>
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<tr>
<td>R0</td>
<td>0.00015625</td>
<td>0.00015625</td>
<td>0.542</td>
<td>3470</td>
<td>0.0446</td>
<td>0.0103</td>
<td>0.0587</td>
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<td>0.00015625</td>
<td>0.537</td>
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<td>0.00062500</td>
<td>0.610</td>
<td>3900</td>
<td>0.0513</td>
<td>0.0283</td>
<td>0.0519</td>
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</tr>
<tr>
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<td>0.00015625</td>
<td>0.00062500</td>
<td>0.614</td>
<td>3930</td>
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<td>0.0275</td>
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<td>0.00031250</td>
<td>0.492</td>
<td>1570</td>
<td>0.0433</td>
<td>0.00792</td>
<td>0.0491</td>
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<td>0.00031250</td>
<td>0.503</td>
<td>1610</td>
<td>0.0452</td>
<td>0.00941</td>
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<td>0.00062500</td>
<td>0.502</td>
<td>1610</td>
<td>0.0431</td>
<td>0.0111</td>
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<td>R7</td>
<td>0.00062500</td>
<td>0.00062500</td>
<td>0.449</td>
<td>718</td>
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<td>0.00540</td>
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<td>0.00062500</td>
<td>0.448</td>
<td>717</td>
<td>0.0399</td>
<td>0.00502</td>
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<td>0.00125000</td>
<td>0.372</td>
<td>298</td>
<td>0.0370</td>
<td>0.00332</td>
<td>0.0385</td>
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<tr>
<td>R10</td>
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<td>0.00125000</td>
<td>0.371</td>
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<tr>
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<td>0.00250000</td>
<td>0.279</td>
<td>112</td>
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<td>0.00272</td>
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<tr>
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<td>0.00500000</td>
<td>0.183</td>
<td>36.7</td>
<td>0.0215</td>
<td>0.00317</td>
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<td>0.01000000</td>
<td>0.103</td>
<td>10.3</td>
<td>0.0132</td>
<td>0.00394</td>
<td>0.0168</td>
<td>5209.60</td>
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<tr>
<td>R14</td>
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<td>0.02000000</td>
<td>0.0547</td>
<td>2.73</td>
<td>0.00822</td>
<td>0.00511</td>
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<td>R15</td>
<td>0.04000000</td>
<td>0.04000000</td>
<td>0.0307</td>
<td>0.767</td>
<td>0.00623</td>
<td>0.00544</td>
<td>0.0113</td>
<td>10240.3</td>
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<tr>
<td>R16</td>
<td>0.08000000</td>
<td>0.08000000</td>
<td>0.0197</td>
<td>0.248</td>
<td>0.00550</td>
<td>0.00612</td>
<td>0.0105</td>
<td>10240.5</td>
<td>32$^2 \times 64$</td>
</tr>
</tbody>
</table>
Coronal heating due to nanoflares?

Parker (1988):

The purpose of the present writing is to emphasize that, collectively, the observations suggest that what we see as the X-ray corona is simply the superposition of a very large number of nanoflares. That is to say, a statistical distribution of nanoflares ranging downward in individual energy from about $10^{27}$ ergs makes up the phenomenon that we call the X-ray corona.

Constraint on energy distribution $dN/dE$ due to nanoflare heating:

If \[ \frac{dN}{dE} \propto E^{-\alpha} \], \[ E_{\text{total}} \sim \int E \frac{dN}{dE} dE \propto E^{2-\alpha} \bigg|^{E_{\text{max}}}_{E_{\text{min}}} \]

Heating dominated by nanoflares if $\alpha > 2$

Heating dominated by large flares if $\alpha < 2$

[Hudson 1991]
Observed solar flare distribution -- energy

• E.g., from Crosby, Aschwanden, & Dennis, Sol. Phys., 1993:

The frequency distribution of the total energy in electrons above 25 keV calculated by integrating the thick-target energy rate over the flare duration.

• Power law distribution: \( \frac{dN}{dE} \sim E^{-1.53} \)
Observed solar flare distribution -- duration

- E.g., from Crosby, Aschwanden, & Dennis, Sol. Phys., 1993:

![Graph showing the frequency distribution of total flare duration](image)

Fig. 7. The frequency distribution of the total flare duration for the same subset as in Figure 2. Note that the power law exhibits a trend to steepen with longer durations such that the slope changes from $-2.17$ to $-2.54$ in the interval from 200 to 3000 s (see Table II).

- Flare duration distribution: \( \frac{dN}{d\tau} \sim \tau^{-2.36} \)
Observed solar flare distribution -- duration

• E.g., from Crosby, Aschwanden, & Dennis, Sol. Phys., 1993:

\[ \frac{dN}{dE} \propto E^{-\alpha} \]

\[ \tau \propto E^\beta \]

Then

\[ \frac{dN}{d\tau} \propto \tau^{-1-(\alpha-1)/\beta} \]

If \( \alpha \sim 1.5 \)

\[ \beta \sim 0.5 \]

\[ \frac{dN}{d\tau} \propto \tau^{-2} \]

Fig. 7. The frequency distribution of the total flare duration for the same subset as in Figure 2. Note that the power law exhibits a trend to steepen with longer durations such that the slope changes from \(-2.17\) to \(-2.54\) in the interval from 200 to 3000 s (see Table II).

• Flare duration distribution: \( \frac{dN}{d\tau} \sim \tau^{-2.36} \), since \( \frac{dN}{dE} \sim E^{-1.53} \Rightarrow \tau \sim E^{0.39} \)
Observed solar flare distribution -- peak

- E.g., from Crosby, Aschwanden, & Dennis, Sol. Phys., 1993:

\[ \frac{dN}{dP} \sim P^{-1.67}, \quad \text{since} \quad \frac{dN}{dE} \sim E^{-1.53} \Rightarrow P \sim E^{0.79} \]
Observed solar flare distribution -- correlation

- E.g., from Crosby, Aschwanden, & Dennis, Sol. Phys., 1993:

\[ P \sim E^{0.79} \quad \text{and} \quad \tau \sim E^{0.39} \Rightarrow P\tau \sim E^{1.18} \]

The \( P\tau \) vs \( E \) scatter plot shows a extremely good power law and can serve as a constrain on the exponents.
Solar flare distribution from dynamical models

- E.g., from Lu & Hamilton, ApJ., 1991:

- Behavior of flares regarded as self-organized criticality (SOC) and described as avalanches in a sandpile model.

- Power law distributions found with power index around 1.4:
  \[
  \frac{dN}{dE} \sim E^{-1.4}
  \]
  \[
  \frac{dN}{dP} \sim P^{-1.8}
  \]

![Graph showing distributions of event energy release, peak flux, and duration for avalanches in a reconnection model.](image)

Fig. 4.—Distributions of event energy release $N(E)$, peak flux $N(P)$, and duration $N(T)$ for avalanches in the reconnection model on a $30^3$ grid. The energy and peak flux distributions are well approximated by a power law over most of the range. The power-law relations $N(E) \propto E^{-1.4}$ and $N(P) \propto P^{-1.8}$ are also shown for comparison. Note that $N(E)$ and $N(T)$ are offset by factors of 100 and 10, respectively.
Heating event distribution in 2D MHD turbulent

- From Dmitruk & Gomez, ApJL 1997:

- Power law distribution:
  \[ P(E) = AE^{-1.5\pm0.2} \]

- Not many results from direct simulations of fluid model, likely due to difficulties in running high-resolution for a long time.

- Simulations done using 2D RMHD with \( \frac{d}{dz} \) terms served as specified random driving.

- 2D is probably too simplistic to describe real corona.

- resolution: \( 96^2 \)
Heating event distribution in 2D MHD turbulent


\[ P(E) = AE^{-1.5 \pm 0.2} \]

\[ \alpha_\tau = 1.9 \pm 0.2. \]

- resolution: \(192^2\)
Heating event distribution in 2D MHD turbulent


![Graph showing dissipation rate over time](image-url)
Heating event distribution in our 3D simulations

- resolution: $256^2 \times 32$, $\eta = \nu = 0.0003125$
Heating event distribution in our 3D simulations

- resolution: $256^2 \times 32$, $\eta = \nu = 0.0003125$
Power law in energy distribution?

- $256^2 \times 32, \eta = \nu = 0.0003125$
- $\alpha = 1.64 \pm 0.12$ by fitting the first 7 points with $dN >= 10$ ($R^2 = 0.97$)
Duration distribution in our 3D simulations

- $256^2 \times 32$, $\eta = \nu = 0.0003125$
Duration distribution in our 3D simulations

\[
\frac{dN}{d\tau} \propto \tau^{-2}
\]

- \(256^2 \times 32, \quad \eta = \nu = 0.0003125\)
Power law in duration distribution?

\[ dN \propto \tau^{-2.28} \]

- \( 256^2 \times 32, \ \eta = \nu = 0.0003125 \)
- \( \alpha = 2.28 \pm 0.24 \) by fitting points with \( dN > 10 \) except the first three (\( R^2 = 0.98 \))
Maybe exponential?

- $256^2 \times 32, \eta = \nu = 0.0003125$
- $\exp(-0.067 \pm 0.0038 \, \tau)$, by fitting points with $dN \geq 10$ ($R^2 = 0.98$)
Correlation between energy and duration

- $256^2 \times 32, \eta = \nu = 0.0003125$


$E \sim \tau^{\gamma_{E\tau}}$, where $\gamma_{E\tau} = 2.02 \pm 0.02$
Peak flux distribution in our 3D simulations

- $256^2 \times 32, \eta = \nu = 0.0003125$

\[
\frac{dN}{dP} \propto P^{-2.08 \pm 0.17}
\]

- Using this and \( \frac{dN}{d\tau} \propto \tau^{-2.28} \), \( \frac{dN}{dE} \propto E^{-1.64} \), expect \( P\tau \propto E^{1.093} \)

--- close to measured value

\[
P\tau \propto E^{1.065 \pm 0.003}
\]

(Note: \( P \propto E^{0.631 \pm 0.005} \), \( \tau \propto E^{0.435 \pm 0.005} \).)
Wait time distribution vs observations

- $256^2 \times 32$, $\eta = \nu = 0.0003125$

- $\alpha_{\tau} = 2.08 \pm 0.17$, $R^2 = 0.98$

- $w$ and $E$ do not appear to be correlated

- $w$ has a similar distribution as $\tau$

Heating event distributions in lower resolutions

\[ \alpha_E = 1.61 \pm 0.15, \quad R^2 = 0.95 \]

\[ \alpha_\tau = 2.36 \pm 0.42, \quad R^2 = 0.91 \]

\[ \alpha_P = 2.067 \pm 0.22, \quad R^2 = 0.95 \]

\[ \alpha_w = 2.19 \pm 0.16, \quad R^2 = 0.98 \]

- \(128^2 \times 32, \quad \eta = \nu = 0.000625\)

\[ \tau \propto E^{0.424 \pm 0.005} \quad P \propto E^{0.643 \pm 0.005} \quad P\tau \propto E^{1.067 \pm 0.003} \]
Heating event distributions in lower resolutions

- $64^2 \times 16, \ \eta = \nu = 0.0025$

\[
\begin{align*}
\tau &\propto E^{0.407 \pm 0.004} \\
P &\propto E^{0.625 \pm 0.004} \\
P\tau &\propto E^{1.032 \pm 0.002}
\end{align*}
\]
Heating event distributions in lower resolutions

- $64^2 \times 16$, $\eta = \nu = 0.005$

\[
\tau \propto E^{0.397 \pm 0.003} \quad P \propto E^{0.636 \pm 0.003} \quad P\tau \propto E^{1.034 \pm 0.001}
\]
Scalings of exponents

- Power law indices in the small resistivity limit are similar to reported results from 2D runs, and are consistent with some observations.
- Higher resolution runs needed to confirm the saturation in the small resistivity limit.
Conclusions

♦ Parker's model of coronal heating has been studied based on a series of 3D RMHD simulations.

♦ Efforts are being made to perform statistical studies based on this large volume of numerical data.

♦ Heating event distributions have been calculated using a method originally applied to 2D simulations. Results are found to be similar, and are consistent with some observations.

♦ Since the energy distribution power law index is significantly less than 2, large heating events are more important to the overall heating in our numerical simulations. If the same is true in the corona, over a large energy range, Parker’s picture of nanoflare heating becomes questionable.