The Effect of Magnetic Turbulence Energy Spectra and Pickup Ions on the Heating of the Solar Wind

C. S. Ng

Geophysical Institute, University of Alaska Fairbanks

A. Bhattacharjee, P. A. Isenberg, D. Munsi, and C. W. Smith

Space Science Center, University of New Hampshire

9th Annual International Astrophysics Conference:
Pickup Ions Throughout the Heliosphere and Beyond
15 March, 2010

Work partially supported by NASA, DOE, and NSF
MHD turbulence

Energy cascade possible only if two Alfvén wave packets propagating in opposite directions collide

Dimensionless parameter: \( \chi = k^{1/2} E_k^{1/2} V_A^{-1} = \nu_k / V_A << 1 \)

- Time scales:
  - Eddy turn-over time \( \tau_N \sim 1 / k \nu_k \)
  - Alfvén time \( \tau_A \sim 1 / k V_A = \chi \tau_N << \tau_N \)
  - Energy cascade time \( \tau_E \sim \tau_A / \chi^2 = \tau_N / \chi >> \tau_N \)

Kolmogorov cascade rate: \( \varepsilon_K \sim \nu_k^2 / \tau_N \sim k^{5/2} E_k^{3/2} \)

IK cascade rate: \( \varepsilon_{IK} \sim \nu_k^2 / \tau_E \sim \varepsilon_K \chi \sim k^3 E_k^2 V_A^{-1} << \varepsilon_K \)

[Iroshnikov 1963, Kraichnan 1965]
Solar wind turbulence model

The steady state solar wind turbulence model developed by [Matthaeus et al. 1994, 1996] and later developments:

Assumptions:

♦ Steady state

♦ Radially expanding solar wind with uniform speed $V_{sw}$

♦ 1D (radial position $r$)

♦ Turbulence characterized by two fields: fluctuation velocity ($Z$) and correlation length ($\lambda$)

♦ Kolmogorov type cascade rate

♦ Solar wind (proton) temperature can be calculated passively
Solar wind turbulence model

The steady state solar wind turbulence model developed by [Matthaeus et al. 1994, 1996] and later developments:

\[
\frac{dZ^2}{dr} = -\frac{AZ^2}{r} - \frac{\alpha Z^3}{\lambda V_{SW}} + \frac{Q}{V_{SW}}
\]

\[
\frac{d\lambda}{dr} = -\frac{C\lambda}{r} + \frac{\beta Z}{V_{SW}} - \frac{\beta\lambda Q}{V_{SW}Z^2}
\]

\[
\frac{dT}{dr} = -\frac{4T}{3r} + \frac{m\alpha Z^3}{3k_B \lambda V_{SW}}
\]

$Z^2$: average turbulence energy with $Z_+^2 = Z_-^2$

$\lambda$: turbulence correlation length

$T$: solar wind proton temperature

$Q$: turbulence generation rate due to pickup ions (more on the form of this operator later)
Solar wind turbulence model vs observations

From [Smith et al. 2001]
Temperature comparison with pickup ions

From [Isenberg et al. 2003]
Solar wind model with IK cascade

[Ng et al., J. Geophys. Res., 115, A02101 (2010)]

The solar wind turbulence model changed to [Matthaeus et al. 1994, Hossain et al. 1995]:

\[
\frac{dZ^2}{dr} = - \frac{AZ^2}{r} - \frac{\alpha Z^4}{\lambda V_{SW} V_A} + \frac{Q}{V_{SW}}
\]

\[
\frac{d\lambda}{dr} = - \frac{C\lambda}{r} + \frac{\beta}{V_{SW}} \left( \frac{Z^4}{V_A} \right)^{1/3} - \frac{\beta \lambda Q}{V_{SW} Z^2}
\]

\[
\frac{dT}{dr} = - \frac{4T}{3r} + \frac{m\alpha Z^4}{3k_B \lambda V_{SW} V_A}
\]

\(Z^2\): average turbulence energy with \(Z_+^2 = Z_-^2\)
\(\lambda\): turbulence correlation length
\(T\): solar wind temperature
\(Q\): turbulence generation rate due to pickup ions (also depends on the turbulence energy spectrum)
Turbulence generation due to pickup ions

Following Isenberg et al. [2003] and Isenberg [2005]

\[ Q = \zeta \frac{V_{SW}^2}{n} \frac{dN}{dt} \]

\[ \frac{dN}{dt} = N_0 \nu_0 (r_E / r)^2 \exp(-L / r) \]

\( V_{SW}^2 / n \): initial kinetic energy per pickup proton

\( N_0 \): neutral hydrogen density at the termination shock

\( \nu_0 \): ionization rate

\( L \): scale of the ionization cavity (6 ~ 8 AU used in calculations)

\( \zeta \): fraction of newly ionized pickup proton energy that generates waves (requires most theoretical considerations)
\[ \xi(\Delta) = 1 - \frac{\Delta + \frac{1}{4} V_{SW}^4 \int_{\Delta}^{1} \nu^4(\mu)S(\mu)d\mu}{\Delta + \frac{1}{2} V_{SW}^2 \int_{\Delta}^{1} \nu^2(\mu)S(\mu)d\mu} \] 

\[ \frac{dv}{d\mu} = \left[ \sum_j \frac{V_j I_j(k_r)}{|\mu v - W_j|} \left( 1 - \frac{\mu V_j}{v} \right) \right] \left[ \sum_j \frac{I_j(k_r)}{|\mu v - W_j|} \left( 1 - \frac{\mu V_j}{v} \right)^2 \right]^{-1} \]

\[ \Delta = Z / 3^{1/2} V_A \quad v(\mu = \Delta) = V_{SW} \quad S(\mu) = |\delta v| \quad S(\mu = \Delta) = 1 \]

\[ k_r = \frac{\Omega}{\mu v - V_j} \quad V_j^3 - \mu v V_j^2 + \mu v V_A^2 = 0 \quad W_j = -\frac{2\mu v V_A^2}{2 V_j^2 - 2\mu v V_j + V_A^2} \]

\[ I(k) = A(r) |k|^{-5/3} \text{ or } I(k) = A(r) |k|^{-3/2} \]

\[ \xi > 0 \text{ if } \frac{v}{V_{SW}} < 1, \quad \xi \to 0 \text{ as } \frac{dv}{d\mu} \to 0 \]

\[ \left[ \text{Isenberg } 2005 \right] \]

\( v \): particle velocity

\( \mu \): \( \cos \theta \)
Cyclotron resonance condition

\[ k_r = \frac{\Omega}{\mu \nu - V_j} \]
\[ \omega(k) = \pm k V_A \sqrt{1 + \frac{\omega}{\Omega}} \]

\( k_1 < k_2 \)

\[ \frac{dv}{d\mu} \propto \sum_j \frac{V_j I_j(k_r)}{|\mu \nu - W_j|} \left( 1 - \frac{\mu V_j}{\nu} \right) \]
\(< 0 \) usually in a turbulence cascade

since \( I(k_1) > I(k_2) \)

\( \xi \) is smaller for the IK spectrum than the Kolmogorov spectrum since \( |dv/d\mu| \) is smaller due to \( I(k_2) \) getting closer to \( I(k_1) \)

[Isenberg 2005]
$\xi$ for Kolmogorov vs IK

![Graphs showing $\Delta$ vs $V_A/V_{SW}$ for Kolmogorov and IK models.](image)
$\zeta$ for Kolmogorov vs IK

[Isenberg 2005]

\[ V_A / V_{SW} = 0.075 \]
Comparisons with observations

cf. [Isenberg et al. 2003]
Comparisons with observations

$Z^2/Z^2(1\text{AU})$

(a) $R \text{ (AU)}$

Kolmogorov without $Q$

Kolmogorov with $Q$

IK with $Q$

IK without $Q$

cf. [Smith et al. 2001]
Comparisons with observations

cf. [Smith et al. 2001]
Comparisons with observations

cf. [Smith et al. 2006]
Conclusion

• Although a solar wind model based on Kolmogorov cascade compares well with proton temperature observations when contributions from pickup ions are included, observations are consistent with IK cascade are less sensitive to the contribution of pickup ions.

• Pickup ions generate less turbulence under IK cascade.

• The prediction of the correlation length based on IK cascade appears to agree better with data than Kolmogorov cascade with pickup ions.

• Results obtained based on parameters/methods used in Smith et al. 2001, Isenberg et al. 2003, and Smith et al. 2006.

• Solar wind heating based on IK cascade is a possible alternative to Kolmogorov based theories and should be investigated more extensively.

• More precise description of how pickup ions generate turbulence might also change the prediction of solar wind temperature, especially in the outer heliosphere.