1 Normally Distributed Random Data

The below figures were created in MATLAB 2007a using normally distributed random data. The Box-Muller algorithm(1) was used to create a normally distributed ensemble with zero mean ($\mu = 0$) and unit variance ($\sigma^2 = 1$) from two sets of uniformly distributed random numbers in the interval $(0, 1]$. Using a linear transform of the form $X = \sigma Z + \mu$, the ensemble was given mean, $\mu = -1$ and standard deviation, $\sigma = 0.5$.

Figure 1. Normally distributed random data for $n = 100$. 
Figure 2. Normally distributed random data for \( n = 1000 \).

Figure 3. Normally distributed random data for \( n = 10000 \).
For a small number of data points \((n)\) the set approximates a normal distribution. Increasing the numbers of data points the ensemble more closely approximates the normal distribution as can be seen in the histograms for \(n = 100, 1000, 10000\).

By increasing the number of data points the Discrete Cumulative Distribution(4), begins to become smoother and can be approximated by

\[
D(x) = \sum_{X \leq x} P(x) \\
\approx \frac{1}{2} \left[ 1 + \text{erf} \left( \frac{x - \mu}{\sigma\sqrt{2}} \right) \right]
\]

where \(P(x)\) is the normal distribution.

2 Uniformly Distributed Random Data

The below figures were created in MATLAB 2007a using uniformly distributed random data centered about zero \((\mu = 0)\) and contained in the interval \([-2, 2]\).

Figure 4. Uniformly distributed random data for \(n = 100\).
Figure 5. Uniformly distributed random data for n = 1000.

Figure 6. Uniformly distributed random data for n = 10000.
Discrete uniformly distributed data has a constant probability. That is for a set of random elements, the \( n^{th} \) element has an equal probability of taking on a value as any other element in the finite set. For a uniform distribution on the interval \([a, b]\), the discrete probability density function is a step function and the discrete cumulative distribution function is a linear line with positive slope.

This can be seen in the above figures. As the size of the ensemble increases, the closer the probability density (histogram) approximates a step function and the discrete cumulative distribution approximates a positive slope line with slope \( m \approx \frac{x-a}{b-a} \), over the interval \([a, b] = [-2, 2]\).

References


