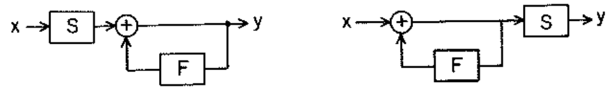
1.21 Show that these two ARMA systems have the same transfer function.




1.22 Following the procedure used in Problem 1.15, compute the 4th forward difference operator.

1.23 Any interpolation scheme must make some assumption about the missing data. If we assume that the data behaves locally like a 3rd-degree polynomial, then the 4th difference operator should annihilate the data at every point. Use the 4th forward difference to find an operator that interpolates the center point from its four neighboring points, two on each side.

1.24 Give an example of an unstable sequence.

1.25 Give an example of a stable causal sequence.

1.26 We would like to make a digital recording of a 3-minute-long stereo music performance. Assume that frequencies exist up to 20 kHz but there are none above that limit. We would like a 96-decibel(db) dynamic range (i.e., each sample will require two 8-bit bytes). What is the minimum sampling rate that we could use? How many bytes of storage will be required to record the program at this minimum sampling rate?

1.27 By expanding $y(x_i + h)$ and $y(x_i - h)$ in a power series about $x_i$, show that $\Delta^2 y = (y_{i+1} - 2y_i + y_{i-1})/h^2$ does equal the second derivative of $y$ at $(x_i)$ plus correction terms of order of $h^2$.

# 2 Sampled Data and the *Z* Transform

Every piece of mathematical machinery must be weighted for the power and insight it produces versus the overhead required to grease and turn its wheels through a given problem area. For studying digital signals and systems, the *Z* transform offers an outstanding price/performance ratio. This amazingly simple transform, with its intimate connection with the all-important convolution operation, requires only a slight mathematical investment in return for a powerful insight into digital, linear shift-invariant systems. Furthermore, it turns out that the *Z* transform is a generalization of the Fourier transform of periodic signals. In this chapter, we will first introduce the *Z* transform and then exploit it in the discussion of the properties of systems and sequencies.

## The *Z* Transform, Polynomial Multiplication, and Convolution

In the previous chapter, we have represented sampled data by writing the values, corresponding to the magnitude of the signal at each time, as an array of numbers. The sequence

$$x(t) = (1, 3, 0, -1, -5)$$

for example, has a value of one at $t = 0$, a value of three at the second time increment, and so on. This sequence may represent a stream of data or the impulse response of an LSI system. The sequence may be infinitely long, as is the case for the impulse response of an IIR system, but for our opening discussion we have a finite sequence in mind, such as the data gathered from any real experiment.

Since the limitation of equally spaced data is fundamental to our discussion, we have introduced in the previous chapter the idea of delay and advance operators. These operators are an important component both of the convolution operation and of computing hardware implemented by shift registers. For this reason, it will behoove us to cast these delay and advance operators into a mathematical scheme. As we will see shortly, by introducing a variable called $Z$, with its powers corresponding to the number of delays or advances, an attractive picture emerges. That is, our example above would be written

$$X(Z) = 1 + 3Z - Z^3 - 5Z^4$$

This polynomial in $Z$ is called the *Z transform* of the sequence $x(t)$. Each power of $Z$ produces one unit of delay. The last term is delayed four time increments from the first term at $t = 0$. Advances are represented by inverse powers of $Z$. If our example sequence were advanced one time increment, the resulting new sequence would become

$$Z^{-1}X(Z) = Z^{-1} + 3 - Z^2 - 5Z^3$$

We must immediately pause to explain that, unfortunately, there are two conventions used for the $Z$ transform. The one we are using was introduced by Laplace and is generally used by physicists and other scientists. Engineers commonly use the negative powers of $Z$ for the delay operators and positive powers for the advance operators. Using Laplace's convention results in writing fewer negative powers because advance operators are in the minority. Switching conventions is clearly a trivial but annoying matter, resulting in sort of an occupational hazard when reading the literature using the opposite definition.

One of the chief properties of $Z$ transforms is that their multiplication is equivalent to the convolution of their corresponding time sequences. This is so because polynomial multiplication automatically collects contributions from factors having a common delay (i.e., the same powers of $Z$). To demonstrate, consider that

$$x_1 = (1, 5)$$
$$x_2 = (1, 2, 3)$$

so that

$$x_1 * x_2 = (1, 7, 13, 15)$$



while the $Z$ transforms of the two sequences are

$$X_1(Z) = 1 + 5Z$$
$$X_2(Z) = 1 + 2Z + 3Z^2$$

giving for their product

$$X_1(Z)X_2(Z) = 1 + 7Z + 13Z^2 + 15Z^3$$

which is the $Z$ transform of the convolution of $x_1(t)$ with $x_2(t)$. We call this equivalence of convolution in the time domain with multiplication in the $Z$ domain the *convolution theorem*. It affords two ways of looking at the same operation, which we will see is extremely valuable. For example, we can now use known properties of polynomials to study LSI systems and sampled time data.

## Factoring *Z* Transforms into Couplets

Perhaps the most important property of a polynomial is stated in the fundamental theorem of algebra: Any polynomial of degree $n$ can be factored into exactly $n$ factors. The importance of this fact, for LSI systems, is that any FIR system can be broken down into a series of simpler systems. For example, if we were studying a system with an impulse response

$$H(t) = (2, -1, -1)$$

whose $Z$ transform is

$$H(Z) = 2 - Z - Z^2$$

the result of this system acting on a sampled data sequence could be accomplished in an equivalent fashion by factoring the polynomial:

$$H(Z) = 2 - Z - Z^2 = (2 + Z)(1 - Z)$$

Since multiplication of these two factors corresponds to convolution in the time domain, these two factors lead to a series action of two, 2-term, impulse response systems as shown in Fig. 2.1. Thus, any sequence with an $n$th-degree $Z$ transform may be written as a product of $n$ factors, which we shall call *couplets*:

$$X(Z) = a_0 + a_1Z + \cdots + a_nZ^n = a_n(Z - r_1)(Z - r_2)\cdots(Z - r_n)$$

displaying the $n$ roots, $r_i$, some of which may be complex. Naturally, in most cases, both a system's impulse response and a data sequence will be real. In this situation, the roots of the polynomial in $Z$ will occur in

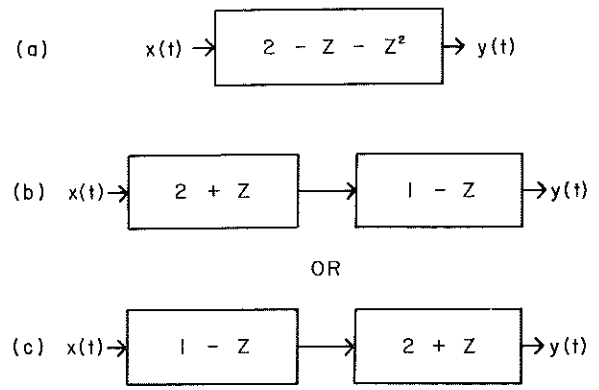



**Figure 2.1** The system with a 3-term response function in (a) can be viewed equivalently as two systems in series, each with a 2-term system response function, as shown in (b) and (c). Since both convolution and multiplication commute, the order is unimportant.

complex pairs. However, remember from our examples in Chapter 1 that there are some cases where we would find a complex data sequence to be convenient. Now we see that the decomposition of a system into its 2-term components will, in general, also lead to complex impulse response functions. Such systems have complex outputs for real inputs.

## Inverse Operators: Stability, Causality, and Minimum Phase

In the previous chapter, we have seen how important inverses are to "undoing" the effects of an LSI operator and how inverses play a critical role in recursive filters and constant-coefficient difference equations. Recall that the meaning of the inverse of an LSI operator is an operator such that

$$A(t) * A(t)^{-1} = \delta$$

where $\delta$ is the spike sequence; it has all zeros except at $t = 0$, where the value is unity. The $Z$ transform of the above equation is simply

$$A(Z)A(Z)^{-1} = 1$$

because of the convolution theorem. Thus, the inverse operator is just

$$A^{-1}(Z) = \frac{1}{A(Z)}$$

So, in the $Z$ domain, writing the inverse of an LSI operator is child's play. But, there are complications. Since the $Z$ transform of any finite sequence



can be written in terms of couplets,

$$A^{-1}(Z) = \frac{1}{a_n(Z - r_1)(Z - r_2)\cdots(Z - r_n)}$$

it seems logical to start our study of inverses by looking at the inverse of a single couplet. Then, we can expand the discussion to include $Z$ transforms of any degree.

Instead of writing $(Z - r)$ for the couplet, we will use $(1 - aZ)$, which only differs by an overall constant. Thus, we are led to consider the inverse

$$(1 - aZ)^{-1} = \frac{1}{1 - aZ}$$

What is the meaning of such an inverse $Z$ transform? Specifically, how is it implemented in a computational scheme? In our discussion on flow graphs we saw one possibility: feedback, that is, recursive schemes involved inverses. The other approach, which we wish to pursue now, is the conversion of this inverse to operations requiring only the input data, avoiding the use of the system's output. This is possible by virtue of the expansion

$$\frac{1}{1 - aZ} = 1 + aZ + (aZ)^2 + (aZ)^3 + \cdots \tag{2.1}$$

which is called the geometric series. It converges absolutely for all $aZ$ such that

$$|aZ| < 1$$

This is a fundamental power series studied in complex analysis; for our purpose we take the result on faith. It follows that $aZ$ is a variable that may take on complex values. Previously, we observed that, in general, we would expect the coefficient $a$ to be some complex number that arises from the roots of a polynomial. This variable $Z$, which was originally designed only to represent delays and advances via its exponent, now demands more serious attention: in order to locate the roots of polynomials in $Z$, we must allow $Z$ to take on complex values.

Returning now to the geometric series expansion, we see that the expression for the inverse will converge on the unit circle if

$$|aZ| < 1 \qquad \text{or} \qquad |a| < 1$$

Under this condition, the inverse can be implemented in a nonrecursive manner, using only the input data. The price paid for avoiding feedback is the requirement for an infinite number of coefficients: an FIR sequence has an IIR inverse. In practice, the terms are truncated when they have diminished to an acceptable level.

Before proceeding, we will belabor the convergence requirement of Eq. (2.1) a bit further. The couplet $(1-aZ)$ has a zero at $Z=Z_0=1/a$. So, we can think of the convergence requirement in three ways: (1) $|a|<1$, that is, (2) the couplet must have the magnitude of the second term less than the magnitude of its first term or (3) the couplet must have its zero outside of the unit circle in the $Z$ plane

$$\left(Z_0=\frac{1}{a}, \qquad |Z_0|>1\right)$$

Such a couplet is called a *minimum phase couplet.* A sequence whose every factor is a minimum phase couplet is a *minimum phase sequence.*

A couplet whose first coefficient has less magnitude than its second is called a *maximum phase couplet*; its zeros are inside the unit circle. A sequence composed of all maximum phase couplets is a *maximum phase sequence*. Sequences containing both minimum and maximum phase couplets are called *mixed phase sequences*. A couplet whose zero lies on the unit circle is a special case, being neither minimum nor maximum phase. Later, when we study the frequency response of couplets, these names will seem more appropriate.

Let us now look at the maximum phase couplet that causes the geometric series of Eq. (2.1) to diverge. Does this mean that it does not have a stable inverse? Actually, it is possible to form a stable inverse by writing

$$\frac{1}{1-aZ}=-\frac{1}{aZ}\left[\frac{1}{1-(1/aZ)}\right]$$
$$=-\frac{1}{aZ}[1+(aZ)^{-1}+(aZ)^{-2}+(aZ)^{-3}+\cdots]$$

where we have used convergence of the geometric series in powers of $1/aZ$ for

$$|aZ|>1 \qquad \text{or} \qquad \frac{1}{|aZ|}<1$$

Accordingly, the maximum phase couplet has a perfectly stable IIR inverse, but note that it requires future values of the input. This is completely unacceptable in some real-time computing situations. For this reason, such an inverse is called physically unrealizable. This term, however, is somewhat misleading because in other applications all the data may be available, such as on a magnetic tape record, and the inverse can be readily applied to future values. Even in the real-time computing environment, the nonrecursive maximum phase inverse may still be useable if a delay in output is tolerable until the series converges to an acceptable level.



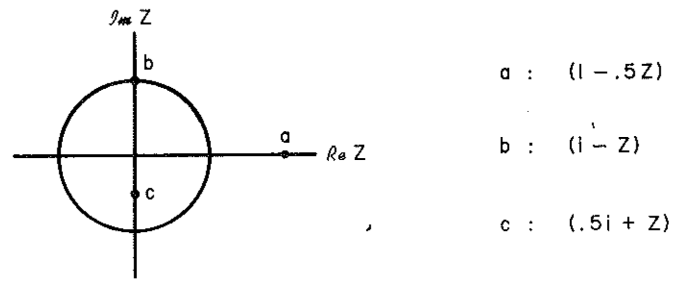



**Figure 2.2** An example of three couplets: a has its zero outside of the unit circle (minimum phase), *b* has its zero right on the unit circle, and c has its zero inside the unit circle (maximum phase). The real part of *Z* is plotted on the *x* axis and its imaginary part is plotted on the *y* axis.

So we have found that, in general, $1/(1-aZ)$ may always be expanded into powers of $Z$ in two ways, into a convergent series or a divergent one. The value of $|a|$ determines which powers of $Z$ will belong to which series. If $|a|<1$, we can expand $1/(1-aZ)$ into either a convergent series in positive powers of $Z$ or a divergent one in negative powers. If $|a|>1$, the reverse is true. The coefficients of these series represent the time domain terms, that is, the inverse $Z$ transform. Because there are always two choices for this expansion, we see that the inverse $Z$ transform is not unique. More often than not we are interested in the convergent, or stable, $Z$ transforms. Figure 2.2 shows examples of three different couplets. The minimum phase couplet has a stable *causal* inverse and the maximum phase couplet has a stable *acausal* (i.e., requires anticipatory data) inverse.

Next, we need to expand the discussion to include $Z$-transform polynomials of degree greater than one. We proceed by example. Consider the sequence $(4,0,-1)$ with $Z$ transform:

$$X(Z)=4-Z^2$$

It has an inverse

$$(4-Z^2)^{-1}=\frac{1}{4-Z^2}=\frac{1}{(2+Z)(2-Z)}=\frac{1/4}{2+Z}+\frac{1/4}{2-Z}$$

The last step is called a partial fraction expansion, which you can verify by adding the last two fractions together. This partial fraction technique allows the geometric series to be used for each inverse couplet. Summing the results for each fraction then gives the expansion of the inverse:

$$(4-Z^2)^{-1}=\frac{1}{8}\left[2+\frac{Z^2}{2}+\frac{Z^4}{8}+\frac{Z^6}{32}+\cdots\right]$$

This partial fraction technique can be applied, in general, by writing unknowns for the numerators of each couplet and solving the resulting equations by standard techniques discussed in algebra books. Likewise, another straightforward method of computing the inverse is to use polynomial division before factoring into couplets. Special treatment is required for multiple poles. Luckily, our concern here is not to develop proficiency in these algebraic gymnastics, but rather we wish to arrive at some important conclusions concerning properties of sequences. Note that the sequence discussed above, $(4, 0, -1)$, factors into two minimum phase couplets. By the partial fraction form of the inverse, we see that the stable inverse of any such minimum phase polynomial will only contain positive powers of $Z$, that is, the inverse will be causal. On the other hand, if the sequence is mixed phase, couplets will occur in the partial fraction sum with both minimum and maximum phase, generating both positive and negative powers of $Z$. Since these terms require anticipatory data, it is acausal: only the pure minimum phase combination (the minimum phase polynomial) has a stable, causal inverse.

Now, we are in a position to consider the most general form of a rational, stable sequence. Its $Z$ transform may be written as a rational fraction of two causal polynomials. Within an overall scale factor, it appears in factored form as

$$S = \frac{(Z-a_1)(Z-a_2)\cdots(Z-a_n)}{(Z-b_1)(Z-b_2)\cdots(Z-b_m)} \tag{2.2}$$

In general, $S$ may have a mixed phase denominator. Stability is assured by expanding the appropriate terms infinitely far into the future and into the past. Thus, with no further restrictions, $S$ represents the most general stable sequence possible. Now, we pursue a classification scheme. If the denominator is pure minimum phase, $S$ is causal and stable. If the numerator is also minimum phase, then we define $S$ to be a minimum phase sequence. This minimum phase sequence will clearly have a causal, stable inverse. This classification scheme is summarized in Fig. 2.3. It is worth emphasizing that not all causal, stable sequences are minimum phase. If $S$ has a minimum phase denominator, but not a minimum phase numerator, $S$ is causal and stable but not minimum phase.

Zeros in the denominator of a fraction cause the fraction to "blow up," that is, to go to infinity. These zeros are called poles of the fraction. They give an alternate succinct statement for the minimum phase condition: a causal stable sequence is minimum phase if its $Z$ transform has no poles or zeros inside the unit circle. This is simply a slightly fancier way of saying the $Z$ transform has no zeros of either its numerator or its denominator inside of the unit circle.



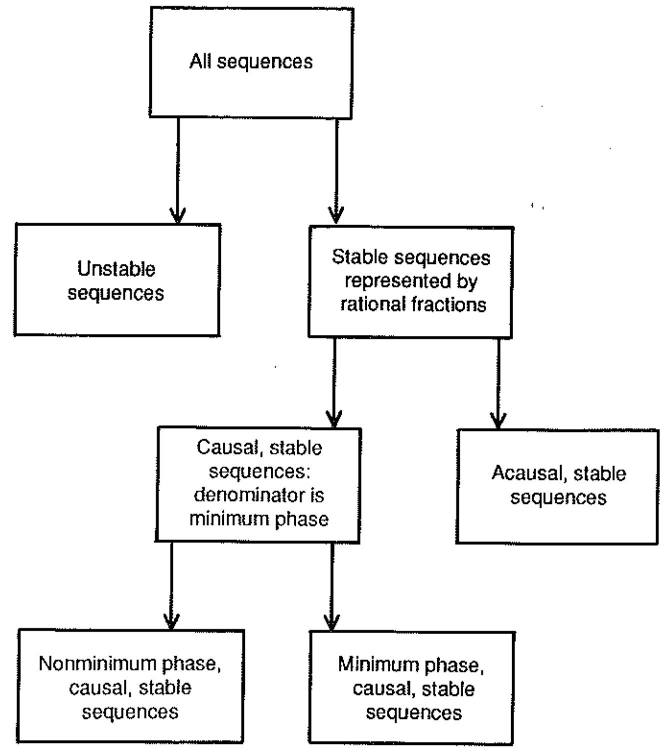



**Figure 2.3** A classification of sequences and systems. The minimum phase sequence is a subclass of causal, stable sequences.

The inverse of $S(Z)$ above is

$$S^{-1} = \frac{(Z-b_1)(Z-b_2)\cdots(Z-b_m)}{(Z-a_1)(Z-a_2)\cdots(Z-a_n)} \tag{2.3}$$

Clearly, if $S$ is minimum phase, so is its inverse. Likewise, $S^{-1}$ is causal and stable. Furthermore, it is only the minimum phase subclass of causal stable sequences that have causal stable inverses. This very important property can be seen easily by comparing Eq. (2.2) and Eq. (2.3): if $S^{-1}$ is to be causal, its denominator, and hence the numerator of $S$, must be minimum phase. Since the denominator of $S$ is already minimum phase (because $S$ is causal), $S$ itself must be minimum phase.

Why have we bothered to define this minimum phase condition? The answer has two parts: the first we will present now, the second will have to wait until we discuss spectra and Fourier transforms. The first important aspect of the minimum phase condition is its relationship to real, physical systems. Consider an example of a one-port, passive, linear, electrical

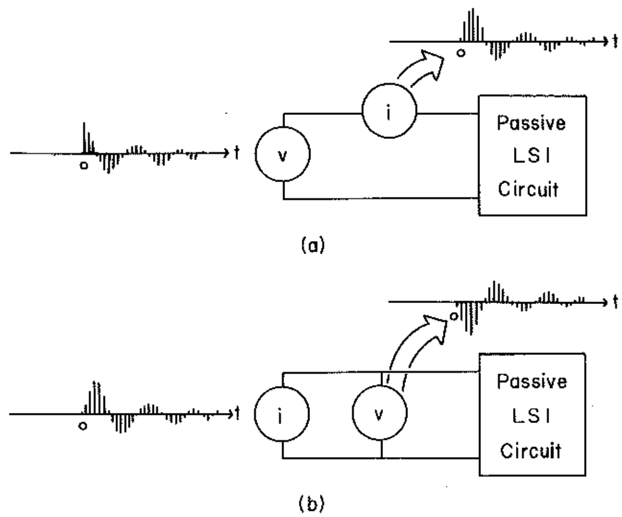



**Figure 2.4** A digitized passive LSI electrical circuit driven in (a) by a voltage and in (b) by a current source. In the first case, the system output is the measured current into the circuit. In the second case, the output is the measured voltage across the circuit.

network. We can choose to excite this network with a voltage source and measure the resulting current, or we can drive it with a current source and measure the resulting voltage. The two situations are depicted in Fig. 2.4. In both cases, the system is causal, that is, no response occurs until there is an excitation present. Normally, you may think of continuous signals in an electrical example like this, but we are primarily interested in sampled data here. Therefore, for the first case, when the excitation is a voltage source, we relate the output of the circuit $i$ to the input by writing

$$i(Z) = v(Z)y(Z)$$

That is, because the system is linear and shift invariant, the output $i$ is related to the input $v$ by a convolution. In the $Z$ domain, the convolution becomes multiplication by the $Z$ transform of $y$, the circuit's admittance. In the second case, when a current source is the excitation, we would likewise write for the output

$$v(Z) = i(Z)z(Z)$$

where $z$ is the $Z$ transform of the circuit's impedance. Clearly, since the circuit is assumed to be passive, both $y$ and $z$ are causal, stable sequences, and furthermore,

$$z(Z) = 1/y(Z)$$



We have seen that the only causal, stable $Z$ transform that has a causal, stable inverse is the minimum phase one. Hence, we arrive at a significant conclusion: a certain real analog system may have a causal inverse demanded by physical considerations. In order to preserve this causal invertibility property when we represent the system digitally, the discrete version must have a minimum phase $Z$ transform. Herein lies one important characteristic of the minimum phase condition—it limits the expected impulse behavior of many systems according to their known physical properties. Generalizing, we can say this: excite any causal LSI system in any way. If (and this is sometimes a big "if") we can argue that the relationship between the input excitation and the output signal is invertible, then the system's response function must be minimum phase.

A very unfortunate, very interesting, and very profound point in digital signal processing is that the minimum phase condition is not necessarily preserved when an analog system is written in discrete form. Furthermore, the minimum phase condition can be erroneously incorporated in the system's response function when it does not belong—one has to be careful. In Problems 2.11 and 2.12, we address a simple analog RC circuit example. Two approaches to finding the digital version of the system's response function are taken; and two different results are obtained. The relationship among the minimum phase condition, physical aspects, and stability (which includes numerical stability of computing schemes) means that the minimum phase condition is going to be an important part of our study in digital signal processing.

This short, but wise, investment in the $Z$ transform will pay out well in the remainder of the book. This is because in any digital application of signal processing, the sequences, systems, and constant-coefficient difference equations can always be represented by a rational fraction of polynomials in $Z$. The $Z$ transform, via the simple geometric series expansion, has shown us how to convert the recursive action of a denominator couplet into an equivalent, although infinitely long, moving average operator. Consequences of this expansion relate the minimum phase condition to a physical aspect—causality. In the next chapters, where we study the frequency response of sequences and systems, the $Z$ transform will again serve as an indispensable framework for our ideas.

## Problems

2.1 Write the $Z$ transforms of the following sequences and factor them into couplets: $(2, -3, -2)$, $(2, -5, 2)$, $(1, -2, 4)$, and $(2, -2, 1)$. Which are minimum phase sequences?

2.2 Express the inverse of $(1, -2)$ as a stable sequence in powers of $Z$.

2.3 Classify the time series $(4, 0, -1)$ and then find its inverse.

2.4 Show that multiplication by $(1-Z)$ is analogous to differentiation in continuous time. Show that dividing by $(1-Z)$ likewise corresponds to integration. What are the limits of this integration?

2.5 A recursive LSI system representing the trapezoid rule of integration can be written

$$y(t) = y(t-1) + \tfrac{1}{2}[x(t) + x(t-1)]$$

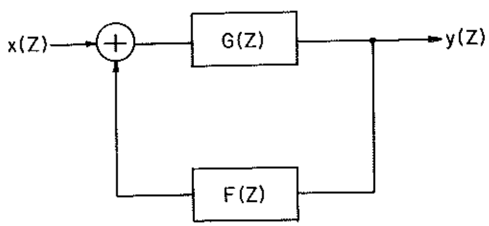
Find the $Z$ transform of the impulse response function of this system. Classify this system according to the location of its poles and zeros in the $Z$ plane.

2.6 A certain nonrecursive LSI system, describing a kind of differentiation, can be written

$$y(t) = \tfrac{1}{2}[x(t+1) - x(t-1)]$$

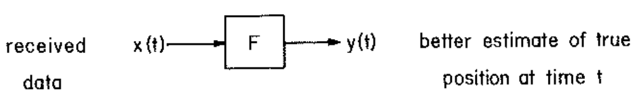
Find its impulse response function in the $Z$ domain and classify it according to its poles and zeros.

2.7 Find the $Z$ transform of the system function for the feedback arrangement shown here in terms of $F(Z)$ and $G(Z)$.




2.8 Specialize Problem 2.7 by setting $G(Z) = 1$. Under what conditions on $F(Z)$ does the system now have a stable output?

2.9 A moving vehicle receives 1-D navigation information at equal time intervals. We would like to smooth this data so as to get a better estimate of the vehicle's true position by using a time domain filter.




We design a recursive filter by reasoning

$$y(t) = y(t-1) + v(t-1)\,\Delta t$$

That is, at time $t$, our improved estimate of position [over simply using the current received information $x(t)$] is the previous estimate



$y(t-1)$ plus the velocity from $t-1$ to $t$ multiplied by the interval $\Delta t$. Estimating

$$v(t-1) = (x(t) - y(t-2))/2\,\Delta t$$

gives us the ARMA filter

$$y(t) = y(t-1) - \tfrac{1}{2}y(t-2) + \tfrac{1}{2}x(t)$$

(a) Show that the above ARMA filter does follow from the reasoning given.

(b) Write down the $Z$ transform of this filter function and classify this digital processing scheme according to the poles and zeros of $F(Z)$.

2.10 Exponentials occur frequently as continuous functions of time in many applications, such as the simple decay of current into a resistive load from a charged capacitor. Sample the continuous exponential

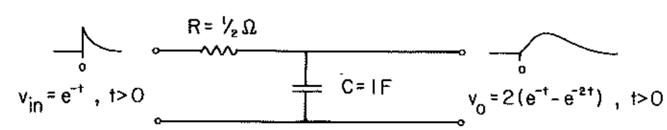
$$x(t) = \begin{cases} 0 & t<0 \\ e^{-t} & t>0 \end{cases}$$

at equal time intervals $\Delta$ to get

$$x(\Delta n) = (e^{-\Delta})^n$$

and use the geometric series to write the $Z$ transform of $x(\Delta n)$.

2.11 The simple RC circuit shown below is excited with a decaying exponential voltage source. The output voltage is the difference of two exponentials as shown. Compute the $Z$ transform of the input voltage, the output voltage, and the transfer function of the circuit. Is the transfer function causal and stable? Is the transfer function minimum phase and, for that matter, would we expect it to be?




2.12 Analog theory tells us that the impulse response of the RC circuit in Problem 2.11 is

$$h(t) = e^{-2t}$$

Digitize this impulse response directly, using the same method of Problem 2.11, showing that this procedure gives a discrete version $H(Z)$ with minimum phase. Which result is better? Is there a correct result?

2.13 Find the inverse of $A = (1 - aZ)$ by expanding it in the geometric series for $|a| < 1$. Then, write a computer program that convolves $A$ with a truncated version of $A^{-1}$. Compute $A * A^{-1}$ for various values of $a$ and for various lengths of $A^{-1}$.

2.14 Repeat Problem 2.13 for values of $|a| > 1$.

2.15 Another way of digitizing an analog system is called step invariance. Even though the $Z$ transform of the step function does not converge, we can still write

$$1/(1 - Z) = 1 + Z + Z^2 + \cdots$$

for the digital form of the step function. Given that a certain analog system has a step response of

$$y(t) = 1 - e^{-t/\tau}$$

digitize the system by computing the ratio of the digitized step function response to the digitized step function input. Where are the poles and zeros of the resulting $H(Z)$?

2.16 Show that the in-place, time-reversed version of an $(n + 1)$-long sequence $A(Z)$ is $Z^n A(1/Z)$.

2.17 Find the $Z$ transform of $\sin \omega t$ and $\cos \omega t$. Comment on the location of their poles and zeros.

2.18 Find the $Z$ transform of $f(t) = e^{-at} \sin \omega t$ for $t > 0$ where $f(t) = 0$ for $t < 0$. Comment on the location of its poles and zeros.

2.19 Sometimes the method of finite differences can be applied to nonlinear differential equations in a straightforward manner. Convert the nonlinear differential equation below to a difference equation using the forward, backward, and central difference operators. What happens to our pole–zero stability analysis now?

$$\frac{dy}{dt} + ay^2 = x(t)$$

# 3 Sinusoidal Response of LSI Systems

All students of the physical sciences learn very early in their studies that sinusoidal functions play a special role in what may seem like the natural scheme of things. Yet, no one has ever seen an actual sine wave; its infinite length precludes the possibility. Of course, we may view a portion of a sine wave, but that is not a pure sine wave; it is too short. Nonetheless, the concept of sinusoidal waves, including their impossible infinite length, gives us valuable insight to certain problems. This insight comes from the fact that sinusoids are the eigenfunctions of LSI systems. Everyone who has studied electrical circuits knows that any arrangement of ideal linear components such as resistors, capacitors, and inductors is an LSI system. In practice, many of these real components approximate an ideal LSI system quite well. If such a system is excited by a sinusoidal source of voltage or current, all resulting voltages and currents vary in a sinusoidal fashion. The frequency of the response is the same as that of the excitation, leaving only the magnitudes and phases as variables for study. This chapter begins our study of this response of LSI systems under sinusoidal excitation; advantages and limitations accompanying this approach will occupy most of the remainder of the book.

The fact that the excitation must be turned on and off means that the source is not a pure sine wave. In some cases, this limitation is satisfactorily dealt with by making measurements after the transient response has died to an acceptable level. On the other hand, the resulting finite data stream creates severe problems in many important cases. This situation applies equally to digital LSI systems. Finite data and operator length will lie at the root of our concern in our study of digital filters, inverse filtering, and spectral estimation.